Anomalous Higgs–top-quark couplings in the MSSM

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The anomalous couplings of the top quark and the Higgs boson have been studied in an effective theory deduced from the minimal supersymmetric extension of the standard model as the heavy fields are integrated out. Constraints on the parameters of the model from the experimental data of $R_b=\Gamma(Z\to b\bar{b})/\Gamma(Z\to\text{hadrons})$ are obtained.

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I. INTRODUCTION

The standard model (SM) has been very successful phenomenologically, but nevertheless it should be considered just as an effective theory valid for physics at the electroweak (EW) scale. In higher-energy regimes new physics beyond the SM must exist. Irrespective of what this new physics might be, it should be able to give a satisfactory answer to the most fundamental open question of electroweak physics, that is, it must explain the origin of the electroweak gauge symmetry breaking [1,2]. In the SM this is arranged through the spontaneous symmetry breaking mechanism by introducing a doublet of scalars with a nonzero vacuum expectation value (VEV). This mechanism, despite its simplicity and economy, has well known problems, which have forced theorists and experimenters to look for new physics beyond the SM. Among the possible ways of extending the SM, supersymmetry is considered as a particularly attractive one. The minimal supersymmetry extension of the standard model (MSSM) provides an appealing solution to the gauge hierarchy problem by guaranteeing the perturbative stability of the theory from the electroweak scale to the Planck scale.

The MSSM contains two complex Higgs doublets, denoted by H_u,H_d and assigned opposite hypercharges $Y_B(H_u)=-Y_B(H_d)=1$. There are altogether four neutral scalar degrees of freedom, three of which correspond to physical scalar fields. In the case where *CP* is conserved one can define two *CP*-even neutral Higgs fields H,h, and one *CP*-odd neutral Higgs field A. The present experimental bounds on the Higgs boson masses set strong restrictions on the parameter space of the *CP*-conserving MSSM [3]. Radiative corrections to the lightest *CP*-even Higgs boson mass have been computed by using the renormalization group equation (RGE) method and diagram technique [4], and the resulting upper bound is 135 GeV, which is not much above the present experimental lower bound of 95 GeV (95% C.L.).

The possibility of *CP* violation makes the situation drastically different. There are three main sources of *CP* violation in the MSSM Lagrangian. The first one is the well known μ parameter of the superpotential, which is in general complex. The second source is constituted by the soft mass terms of the $SU(3)\times SU(2)\times U(1)$ gauginos. The third source is the phases of the soft supersymmetry (SUSY) breaking mass terms of scalar fermions and of the soft trilinear couplings, which are represented by the matrices $\mathbf{m}^2_{Q,U,D,L,R}$ and $\mathbf{A}_{U,D,E}$, respectively. Actually, only the off-diagonal elements of the soft mass matrices can be complex due to the Hermiticity of these matrices. The matrices $\mathbf{A}_{U,D,E}$, in contrast, can have complex phases in their diagonal elements also [5]. Not all the phases of these soft SUSY breaking parameters are physical and lead to the violation of *CP* parity. The physical *CP* phases are restricted by experimental observations, the most rigorous constraints originating from measurements of the electron and neutron electric dipole moments (EDMs). The present upper limits for these EDMs are $d_e<4.3\times10^{-27}\ e$ cm [6] and $d_n<6.5\times10^{-26}\ e$ cm [7], respectively. The EDM of H_g^{199} has also been quite accurately measured, the present upper limit being $d_{H_g^{199}}<9.0\times10^{-28}\ e$ cm [8].

It has been demonstrated that the MSSM can be consistent with these constraints in some regions of the parameter space when suitable cancellations between different contributions occur [9] or when *CP* violation effects are associated with the third generation of squarks only [10]. The mixing of neutral Higgs bosons in the latter scenario is analyzed in [11–14]. It is found that the *CP*-violating phases and large Yukawa couplings of the third generation fermions can lead to large mixings among the neutral Higgs bosons as a consequence of radiative effects. These mixings can drastically change the couplings between the neutral Higgs bosons and quarks and between the neutral Higgs bosons and gauge bosons, as well as the self-couplings of the Higgs boson fields. One consequence of this is that the experimental lower bound on the lightest neutral Higgs boson mass is relaxed to 60 GeV, while the predicted upper limit for the lightest Higgs boson mass remains about 135 GeV.

If the new physics scale is much higher than the EW scale, one would have at the EW scale a great number of higher-dimensional operators $\mathcal{O}_i$ [$\dim(\mathcal{O}_i)>4$] induced by the beyond-the-SM physics [15–18]. The resulting effective Lagrangian is of the general form

$$\mathcal{L}_{eff}=\mathcal{L}_0+\frac{1}{\mu_{NP}^2}\sum_i C_i\mathcal{O}_i+\mathcal{O}\left(\frac{1}{\mu_{NP}^4}\right). \tag{1}$$

Here $\mathcal{L}_0$ is the SM Lagrangian, C_i are Wilson coefficients,

and μ_{NP} is the energy scale of new physics. The Wilson coefficients are in general dependent on the new energy scale, but in addition to this all the higher-dimensional operators in $\mathcal{L}_{eff}$ have a common suppression factor $1/\mu_{NP}^2$.

In this paper we shall study anomalous couplings (to use the terminology of [19,20]), i.e., couplings not present in the SM Lagrangian $\mathcal{L}_0$, between the lightest neutral Higgs scalar (h) and the top quark induced by the new physics of the MSSM. We assume that the other Higgs bosons, as well as all supersymmetric particles, are much heavier than the lightest neutral Higgs particle, so that the corresponding fields can be integrated out. A well known fact is that the masses of the other two neutral Higgs bosons are approximately equal to that of the charged Higgs boson (H^+) under the condition $m_{H^+} \gg m_h$, and hence one can consider the lighter Higgs doublet as the SM Higgs field and integrate out the heavier Higgs doublet.

Our presentation is organized as follows. In Sec. II, the notation adopted in this work is introduced. In Sec. III we shall describe the method of obtaining the Wilson coefficients by integrating out the heavy degrees of freedom in the full theory. A numerical analysis of the constraints on the parameter space from the present experiments, especially by the R_b data, is given in Sec. IV. Section V summarizes our results. Some lengthy formulas, such as the expressions for the Wilson coefficients and the loop integral functions, are collected in the Appendixes.

II. PRELIMINARIES

The most general gauge invariant superpotential, which retains all the conservation laws of the SM, is given by

$$\mathcal{W} = \mu \epsilon_{ij} \hat{H}_u^i \hat{H}_d^j + \epsilon_{ij} h_L^{JI} \hat{H}_d^i \hat{L}_j^I \hat{R}^J + \epsilon_{ij} h_D^{JI} \hat{H}_d^i \hat{Q}_j^I \hat{D}^J + \epsilon_{ij} h_U^{JI} \hat{H}_u^i \hat{Q}_j^I \hat{U}^J. \tag{2}$$

Here $\hat{H}_u, \hat{H}_d$ are the two Higgs superfield doublets, $\hat{Q}^I$ and $\hat{L}^I$ are the doublets of quark and lepton superfields, and $\hat{U}^I$, $\hat{D}^I$, and $\hat{R}^I$ are the singlet superfields of u- and d-type quarks and charged leptons, respectively [$I = 1,2,3$ is the generation index, $i,j = 1,2$ are $SU(2)$ indices]. The Yukawa coupling constants are denoted by $h_L, h_{U,D}$. The breaking of supersymmetry happens through the so-called soft terms, which are in the most general case given by

$$\begin{aligned}\mathcal{L}_{soft} = & - m_{H_u}^2 H_u^{i*} H_u^i - m_{H_d}^2 H_d^{i*} H_d^i - m_{L^{IJ}}^2 \tilde{L}_i^{I*} \tilde{L}_i^J - m_{R^{IJ}}^2 \tilde{R}^{I*} \tilde{R}^J \\ & - m_{Q^{IJ}}^2 \tilde{Q}_i^{I*} \tilde{Q}_i^J - m_{U^{IJ}}^2 \tilde{U}^{I*} \tilde{U}^J - m_{D^{IJ}}^2 \tilde{D}^{I*} \tilde{D}^J \\ & + (m_1 \lambda_B \lambda_B + m_2 \lambda_A^i \lambda_A^i + m_3 \lambda_G^a \lambda_G^a + \text{H.c.}) \\ & + [\epsilon_{ij} m_{H_{12}}^2 H_u^i H_d^j + \epsilon_{ij} A_L^{JI} H_d^i \tilde{L}_j^I \tilde{R}^J + \epsilon_{ij} A_D^{JI} H_d^i \tilde{Q}_j^I \tilde{D}^J \\ & + \epsilon_{ij} A_U^{JI} H_u^i \tilde{Q}_j^I \tilde{U}^J + \text{H.c.}].\end{aligned} \tag{3}$$

Here $\lambda_G^a (a = 1,2,\ldots,8)$, $\lambda_A^i (i = 1,2,3)$, and λ_B denote the $SU(3)$, $SU(2)$, and $U(1)$ gauginos, respectively, and $A_{U,D,L}^{IJ}$ are coupling constants of the unit of mass.

Let us define the scalar doublets Φ and Φ_H as follows:

$$\begin{pmatrix} \Phi \\ \Phi_H \end{pmatrix} = \begin{pmatrix} c_\beta & -s_\beta \\ s_\beta & c_\beta \end{pmatrix} \begin{pmatrix} \tilde{H}_d \\ H_u \end{pmatrix}, \tag{4}$$

where $\tilde{H}_d = i\sigma_2 H_d^*$ and $c_\beta = \cos\beta, s_\beta = \sin\beta$ with $\tan\beta = v_u/v_d$, the ratio of the VEVs of H_u, H_d. With this definition Φ is identified as the SM Higgs doublet, consisting of Goldstone bosons and a physical neutral Higgs field. More explicitly, one can write the two Higgs doublets as

$$\Phi = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(v + H_1^0 + iG^0) \end{pmatrix}, \quad \Phi_H = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}(H_2^0 + iA) \end{pmatrix}, \tag{5}$$

where G^0, G^+ denote the Goldstone bosons, H_1^0 and H_2^0 are the neutral Higgs fields, H^+ and A are the physical charged Higgs bosons and CP-odd neutral Higgs bosons, respectively, and $v = \sqrt{v_u^2 + v_d^2} = 246$ GeV. At the electroweak scale, the two physical CP-even neutral Higgs fields are obtained through the mixing between the fields H_1^0 and H_2^0. The masses of the physical Higgs bosons are given by

$$\mathbf{m}_{even}^2 = \begin{pmatrix} m_Z^2 (s_\beta^2 - c_\beta^2)^2 & 2m_Z^2 s_\beta c_\beta (c_\beta^2 - s_\beta^2) \\ 2m_Z^2 s_\beta c_\beta (c_\beta^2 - s_\beta^2) & \frac{m_{H_{12}}^2}{s_\beta c_\beta} + 4m_Z^2 s_\beta^2 c_\beta^2 \end{pmatrix}$$

$$[\text{in the basis } (H_1^0,\ H_2^0)^{\mathrm{T}}],$$

$$m_A^2 = \frac{m_{H_{12}}^2}{s_\beta c_\beta},$$

$$m_{H^+} = \frac{m_{H_{12}}^2}{s_\beta c_\beta} + m_W^2. \tag{6}$$

In the limit $m_H^2 \gg m_W^2$ the two doublets Φ and Φ_H decouple, the former remaining light and the latter being associated with a large mass $m_H^2 = m_{H_{12}}^2 / s_\beta c_\beta$.

In the following, we will use the four-component spinor representation for fermions. From the two-component Weyl spinors ψ_{Q_I}, ψ_{U_I}, ψ_{D_I}, ψ_{H_u}, and ψ_{H_d}, we form the following four-component Dirac fermions:

$$q_L^I=\begin{pmatrix}\psi_{Q_I}\\ 0\end{pmatrix},\quad u_R^I=\begin{pmatrix}0\\ \overline{\psi}_{U_I}\end{pmatrix},$$

$$d_R^I=\begin{pmatrix}0\\ \overline{\psi}_{D_I}\end{pmatrix},\quad \psi_H=\begin{pmatrix}\psi_{H_u}\\ \overline{\tilde{\psi}}_{H_d}\end{pmatrix}, \tag{7}$$

with $\tilde{\psi}_{H_d}=(i\sigma^2)\psi_{H_d}$. Similarly, for the $SU(3)\times SU(2)\times U(1)$ gauginos $\lambda_G^a,\lambda_A^i,\lambda_B$ we can define the following four-component Majorana spinors:

$$\psi_G^a=\begin{pmatrix}i\lambda_G^a\\ -i\overline{\lambda}_G^a\end{pmatrix},\quad \psi_A^i=\begin{pmatrix}i\lambda_A^i\\ -i\overline{\lambda}_A^i\end{pmatrix},$$

$$\psi_B=\begin{pmatrix}i\lambda_B\\ -i\overline{\lambda}_B\end{pmatrix}. \tag{8}$$

Diagonalizing the soft mass terms is done with the help of the sfermion mixing matrices $\mathcal{Z}_{Q,U,D}$ defined as

$$\mathcal{Z}_Q^\dagger \mathbf{m}_Q^2 \mathcal{Z}_Q=\hat{\mathbf{m}}_Q^2,$$

$$\mathcal{Z}_U^\dagger \mathbf{m}_U^2 \mathcal{Z}_U=\hat{\mathbf{m}}_U^2,$$

$$\mathcal{Z}_D^\dagger \mathbf{m}_D^2 \mathcal{Z}_D=\hat{\mathbf{m}}_D^2, \tag{9}$$

where the matrices $\hat{\mathbf{m}}_{Q,U,D}^2$ on the right-hand side are diagonal.

Finally, we will benefit in our calculations from the following rearrangement identities of the $SU(2)$ group indices:

$$\mathbf{1}_{\alpha\alpha'}\otimes\mathbf{1}_{\beta\beta'}=\frac{1}{2}\left\{\mathbf{1}_{\alpha\beta}\otimes\mathbf{1}_{\beta'\alpha'}+\sum_a \sigma^a_{\alpha\beta}\otimes\sigma^a_{\beta'\alpha'}\right\},$$

$$\sigma^a_{\alpha\alpha'}\otimes\sigma^a_{\beta\beta'}=\frac{1}{2}\left\{\mathbf{31}_{\alpha\beta}\otimes\mathbf{1}_{\beta'\alpha'}-\sum_a \sigma^a_{\alpha\beta}\otimes\sigma^a_{\beta'\alpha'}\right\},$$

$$\sum_{a,b}\epsilon_{abc}\epsilon_{abd}=2\delta_{cd}. \tag{10}$$

III. THE HIGGS-BOSON–TOP-QUARK ANOMALOUS COUPLINGS

In this section we shall discuss the anomalous couplings of the top quark and Higgs bosons. Considering the suppression of the new physics energy scale, we just keep operators up to dimension 6 in the effective Lagrangian Eq. (1). The top-quark–Higgs-boson anomalous couplings of interest can be classified into three types: the anomalous couplings involving a left-handed quark, the right-handed top quark, and Higgs boson ($\mathcal{O}_{tq\Phi}$), the couplings between the Higgs boson and a left-handed quark ($\mathcal{O}_{q\Phi}$), and the couplings between the Higgs boson and the right-handed top quark ($\mathcal{O}_{t\Phi}$). After the EW symmetry breaking, these operators not only produce corrections to the effective couplings $Wt\bar{b}$, $Xt\bar{t}$, $Xb\bar{b}$ ($X=\gamma, Z, H$), but also induce anomalous couplings such as $\gamma Ht\bar{t}$, $ZHt\bar{t}$. All those effects may be detectable at the Next Linear Collider and at the Fermilab Tevatron.

In the following subsections we will give the explicit expressions for the contributions of supersymmetric particles and the heavy Higgs boson doublet to the effective operators mentioned above by deriving the relevant Wilson coefficients. We will give our results in terms of the following loop integral functions:

$$\mathcal{B}^i_{j,k}(x_a,x_b)=\frac{(4\pi)^2}{i}\int\frac{d^4q}{(2\pi)^4}\frac{(q^2)^i}{(q^2-x_a)^j(q^2-x_b)^k},$$

$$\mathcal{C}^i_{jkl}(x_a,x_b,x_c)=\frac{(4\pi)^2}{i}\int\frac{d^4q}{(2\pi)^4}\times\frac{(q^2)^i}{(q^2-x_a)^j(q^2-x_b)^k(q^2-x_c)^l},$$

$$\mathcal{D}^i_{jklm}(x_a,x_b,x_c,x_d)=\frac{(4\pi)^2}{i}\int\frac{d^4q}{(2\pi)^4}\times\frac{(q^2)^i}{(q^2-x_a)^j(q^2-x_b)^k(q^2-x_c)^l(q^2-x_d)^m}. \tag{11}$$

The explicit expressions of these are given in Ref. [21].

A. The anomalous couplings $\mathcal{O}_{tq\Phi}$

This class of operators includes the *CP*-even operators

$$\mathcal{O}_{tq\Phi 1}=(\Phi^\dagger\Phi)(\bar{q}_L t_R\tilde{\Phi}+\tilde{\Phi}^\dagger\bar{t}_R q_L),$$

$$\mathcal{O}_{tq\Phi 2}=\bar{q}_L(D_\mu t_R)D^\mu\tilde{\Phi}+(D_\mu\tilde{\Phi})^\dagger\overline{(D^\mu t_R)}q_L,$$

$$\mathcal{O}_{tq\Phi 3}=\overline{(D_\mu q_L)}(D^\mu t_R)\tilde{\Phi}+\tilde{\Phi}^\dagger\overline{(D_\mu t_R)}(D^\mu q_L),$$

$$\mathcal{O}_{tq\Phi 4}=\overline{(D_\mu q_L)}t_R(D^\mu\tilde{\Phi})+(D_\mu\tilde{\Phi})^\dagger\bar{t}_R(D^\mu q_L),$$

$$\mathcal{O}_{tq\Phi 5}=i\overline{(D_\mu q_L)}\sigma^{\mu\nu}t_R(D_\nu\tilde{\Phi})+i(D_\nu\tilde{\Phi})^\dagger\bar{t}_R\sigma^{\mu\nu}(D_\mu q_L), \tag{12}$$

where the covariant derivative D_μ is given by $D_\mu=\partial_\mu-(i/2)g_3T^AG_\mu^A-(i/2)g_2\sigma^aW_\mu^a-(i/2)g_1Y_BB_\mu$. The *CP*-odd counterparts of these operators are

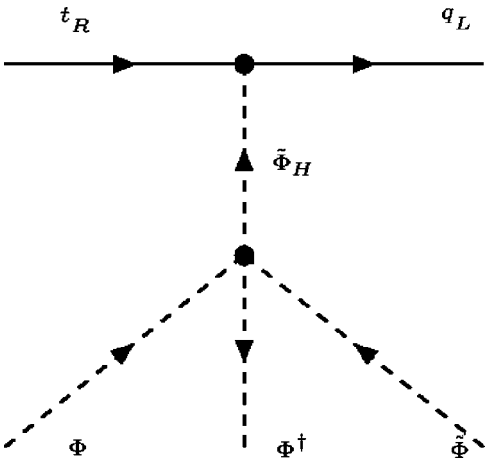

FIG. 1. The Feynman diagrams inducing nontrivial contributions to the Wilson coefficients of the operators $\mathcal{O}_{tq\Phi 1}$ and $\mathcal{O}_{tq\Phi 6}$.

$$\mathcal{O}_{tq\Phi 6}=(\Phi^\dagger\Phi)(\bar{q}_L t_R\tilde{\Phi}-\tilde{\Phi}^\dagger\bar{t}_R q_L),$$

$$\mathcal{O}_{tq\Phi 7}=\bar{q}_L(D_\mu t_R)D^\mu\tilde{\Phi}-(D_\mu\tilde{\Phi})^\dagger\overline{(D^\mu t_R)}q_L,$$

$$\mathcal{O}_{tq\Phi 8}=\overline{(D_\mu q_L)}(D^\mu t_R)\tilde{\Phi}-\tilde{\Phi}^\dagger\overline{(D_\mu t_R)}(D^\mu q_L),$$

$$\mathcal{O}_{tq\Phi 9}=\overline{(D_\mu q_L)}t_R(D^\mu\tilde{\Phi})-(D_\mu\tilde{\Phi})^\dagger\bar{t}_R(D^\mu q_L),$$

$$\mathcal{O}_{tq\Phi 10}=i\overline{(D_\mu q_L)}\sigma^{\mu\nu}t_R(D_\nu\tilde{\Phi})-i(D_\nu\tilde{\Phi})^\dagger\bar{t}_R\sigma^{\mu\nu}(D_\mu q_L). \tag{13}$$

For the *CP*-even operator $\mathcal{O}_{tq\Phi 1}$ and for the corresponding *CP*-odd operator $\mathcal{O}_{tq\Phi 6}$, nonzero contributions to the Wilson coefficients originate from the Feynman diagrams shown in Fig. 1, and they are given by

$$C_{tq\Phi 1}=\frac{1}{2}(g_1^2+g_2^2)\mathrm{Re}(\mathbf{h}_U^{33})\frac{s_\beta c_\beta^2(c_\beta^2-s_\beta^2)}{x_H},$$

$$C_{tq\Phi 6}=i\frac{1}{2}(g_1^2+g_2^2)\mathrm{Im}(\mathbf{h}_U^{33})\frac{s_\beta c_\beta^2(c_\beta^2-s_\beta^2)}{x_H}, \tag{14}$$

where $x_H=m_H^2/\mu_{NP}^2$. In the full theory, the Feynman diagrams that induce the anomalous couplings $\mathcal{O}_{tq\Phi 1},\mathcal{O}_{tq\Phi 6}$ should also include diagrams involving virtual SM fields. However, these diagrams have no contribution to the Wilson coefficients after matching the effective Lagrangian Eq. (1) with the Lagrangian of the full theory (MSSM) (see below for more details).

For the *CP*-even anomalous operators $\mathcal{O}_{tq\Phi i}$ $(i=2,3,4,5)$ and the *CP*-odd anomalous operators $\mathcal{O}_{tq\Phi i}$ $(i=7,8,9,10)$, the derivation of the Wilson coefficients leads to relatively tedious calculations. In Fig. 2 we show the Feynman diagrams, which induce nontrival contributions to the Wilson coefficients after matching the amplitude of the effective theory with that of the MSSM. In these diagrams, the black blobs represent the self-energy diagrams of $\bar{q}_L q_L$, $\bar{t}_R t_R$, and $\Phi^\dagger\Phi$ (Φ_H) (Fig. 3).

The matching procedure to which we refer above is extensively applied in the derivation of the effective Lagrangian in hadron physics, especially in the application of the effective Lagrangian to the rare B decay [22]. The main idea of this procedure is the following. We derive the amplitude corresponding to the relevant Feynman diagrams both in the full theory and in the effective theory. In both derivations we keep only the momenta p_i of external particles to the second order. Through a comparison of the amplitudes of the full theory and the effective theory we then obtain the Wilson coefficients of interest.

For a demonstration, let us consider the first diagram of Fig. 2. In the full theory we can write the amplitude corresponding to this diagram as

$$\begin{aligned}A_{2(1)}^{FT}(p,q)=&-\frac{i}{(4\pi)^2}s_\beta c_\beta^2(\mathbf{h}_D^\dagger\mathbf{h}_D\mathbf{h}_U^\dagger)_{33}\Bigg\{\left[\Delta+1+\ln\frac{\mu_{NP}^2}{m_H^2}\right]\\&\times(\bar{q}_L\tilde{\Phi})t_R-\frac{1}{2m_H^2}\left(1+\ln\frac{m_q^2}{m_H^2}\right)(\bar{q}_L\tilde{\Phi})q^2t_R\\&+\frac{1}{2m_H^2}(\bar{q}_L\tilde{\Phi})(p+q)^2t_R-\frac{1}{2m_H^2}(\bar{q}_L\tilde{\Phi})q\\&\times\cdot(p+q)t_R+\frac{1}{4m_H^2}(\bar{q}_L\tilde{\Phi})[\not{p},\not{q}]t_R\Bigg\}.\end{aligned} \tag{15}$$

Here $\Delta=1/\epsilon-\gamma_E+\ln 4\pi$ denotes the ultraviolet divergence ($D=4-2\epsilon$ is the time-space dimension in the dimensional regularization scheme), μ_{NP} is the scale of new physics, and p and q denote the four-momenta of the external particles t_R and Φ, respectively. In the full theory the light fields and the heavy fields coexist in the Lagrangian. When the heavy degrees of freedom are integrated out and the light fields are treated as massless, infrared divergences are encountered. They are regulated by the parameter m_q.

The amplitude of the corresponding Feynman diagram in the effective theory, presented in Fig. 4, is given by

$$\begin{aligned}A_{2(1)}^{ET}(p,q)=&-\frac{i}{2(4\pi)^2m_H^2}s_\beta c_\beta^2(\mathbf{h}_D^\dagger\mathbf{h}_D\mathbf{h}_U^\dagger)_{33}\\&\times\left(\Delta-\frac{1}{2}+\ln\frac{\mu_{NP}^2}{m_q^2}\right)(\bar{q}_L\tilde{\Phi})q^2t_R.\end{aligned} \tag{16}$$

In the operators of the effective theory, only the light fields exist, and the Wilson coefficients do not depend on their masses. As in the full theory, an infrared divergence emerges here, and it is also regularized by the parameter m_q. As expected, the infrared divergence appearing in the effective theory is the same as that appearing in the full theory. By matching the amplitudes Eq. (16) and Eq. (15), one gets rid of the infrared divergence. After this matching step, we can present the amplitude in its final form:

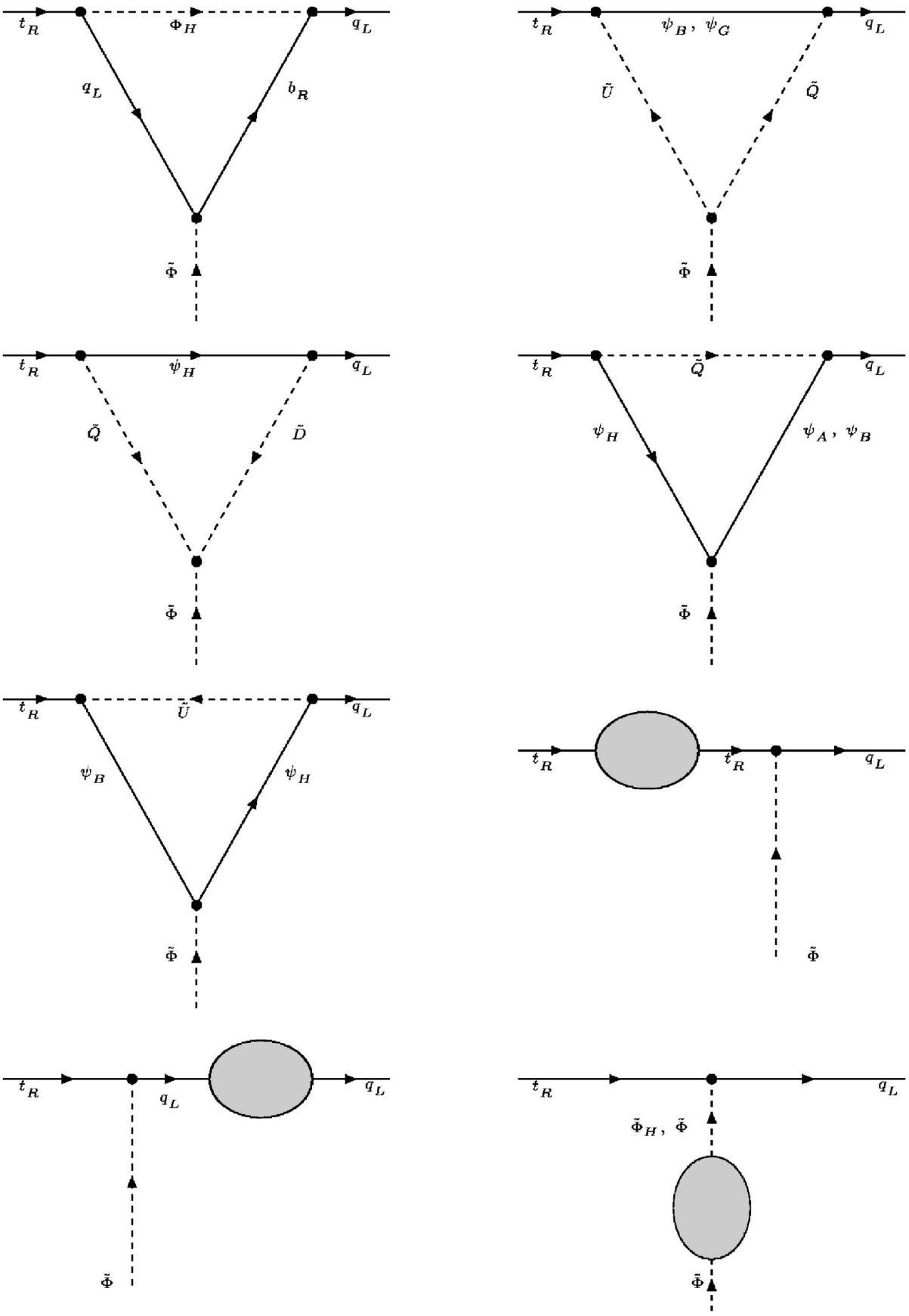

FIG. 2. The Feynman diagrams inducing nontrivial contributions to the Wilson coefficients of the operators $\mathcal{O}_{tq\Phi i}$ ($i = 2, \ldots, 5, 7, \ldots, 10$) in the full theory.

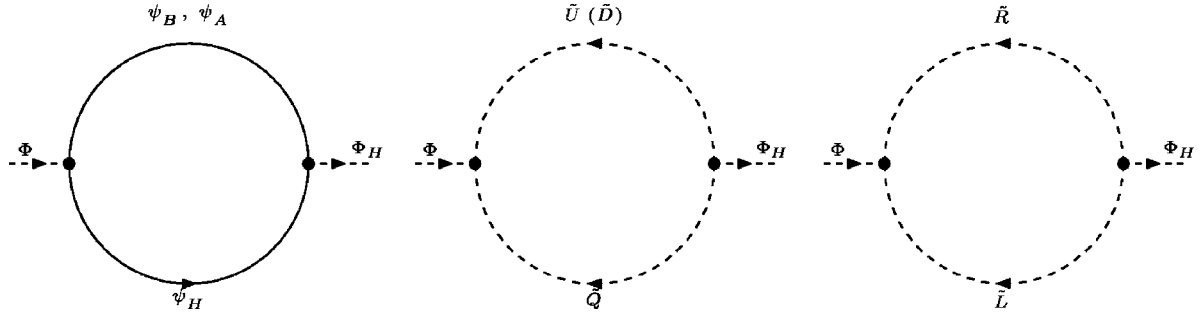

FIG. 3. The Higgs self-energy diagrams that induce nonzero contributions to the Wilson coefficients of operators Eq. (12) and Eq. (13).

$$A_{2(1)}(p,q) = -\frac{i}{(4\pi)^2} s_\beta c_\beta^2 (\mathbf{h}_D^\dagger \mathbf{h}_D \mathbf{h}_U^\dagger)_{33} \left\{ \left[\Delta + 1 + \ln \frac{\mu_{NP}^2}{m_H^2} \right] \times (\bar{q}_L \tilde{\Phi}) t_R + \frac{1}{2m_H^2} \left(1 + \ln \frac{\mu_{NP}^2}{m_H^2} \right) (\bar{q}_L \tilde{\Phi}) p \cdot q t_R - \frac{1}{2m_H^2} \left(1 + \ln \frac{\mu_{NP}^2}{m_H^2} \right) (\bar{q}_L \tilde{\Phi}) q \cdot (p+q) t_R + \frac{1}{2m_H^2} (\bar{q}_L \tilde{\Phi}) p \cdot (p+q) t_R - \frac{1}{4m_H^2} (\bar{q}_L \tilde{\Phi}) \times [\not{q}, \not{p}] t_R \right\}. \quad (17)$$

The first term in parentheses of Eq. (17) contributes to the renormalization of the Yukawa couplings $\mathbf{h}_U$, and it is irrelevant to our present discussion, taking into account the approximation level we work on. For those diagrams where the inner lines are supersymmetry particles, the Wilson coefficients of the anomalous couplings can be directly read from the amplitudes, because we integrate out all the supersymmetry fields in the effective theory.

Now we will turn to showing how to obtain the contributions of the self-energy diagrams to the anomalous couplings. As mentioned above, there are three possible self-energy diagrams that contribute to the coefficients indirectly, namely, the self-energy corrections to the $\bar{q}_L q_L$, $\bar{t}_R t_R$, and Higgs doublet currents. For a fermion, the renormalized fields are defined by

$$f_{L,i}^0 = Z_{L,ij}^{1/2} f_{L,j},$$
$$f_{R,i}^0 = Z_{R,ij}^{1/2} f_{R,j}, \quad (18)$$

where i,j are generation indices, $f_{L,i}^0, f_{R,i}^0$ are the left- and right-handed bare fields, respectively, $f_{L,i}, f_{R,i}$ are the corresponding renormalized fields, and $Z_{L,R}$ are the wave function renormalization constants. Ignoring the fermion masses, we can write down the counterterms for the fermions in Eq. (18) as follows:

$$\Sigma_{ij}^{L,c}(p) = (Z_{L,iI}^{\dagger 1/2} Z_{L,Ij}^{1/2} - \delta_{ij}) \not{p} = \frac{1}{2} (\delta Z_{L,ij}^\dagger + \delta Z_{L,ij}) \not{p},$$
$$\Sigma_{ij}^{R,c}(p) = (Z_{R,iI}^{\dagger 1/2} Z_{R,Ij}^{1/2} - \delta_{ij}) \not{p} = \frac{1}{2} (\delta Z_{R,ij}^\dagger + \delta Z_{R,ij}) \not{p}, \quad (19)$$

where p denotes the external momentum of the fermion. In the full theory, we express the bare self-energy of the fermions as

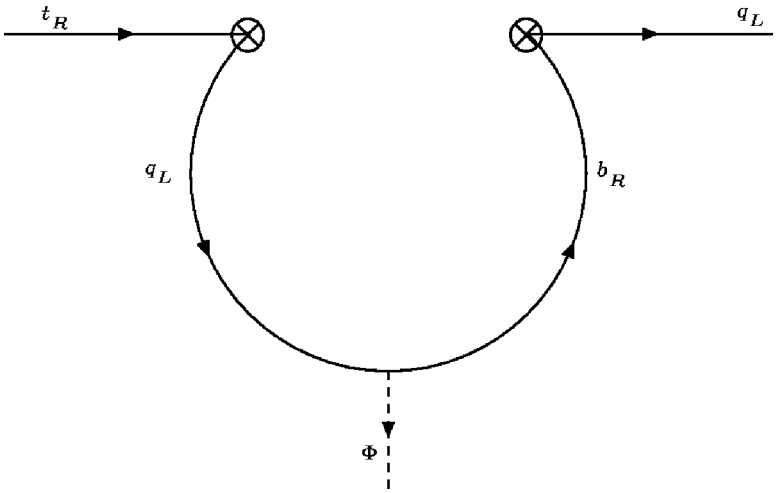

FIG. 4. The Feynman diagram corresponding to the first diagram of Fig. 2 in the effective theory.

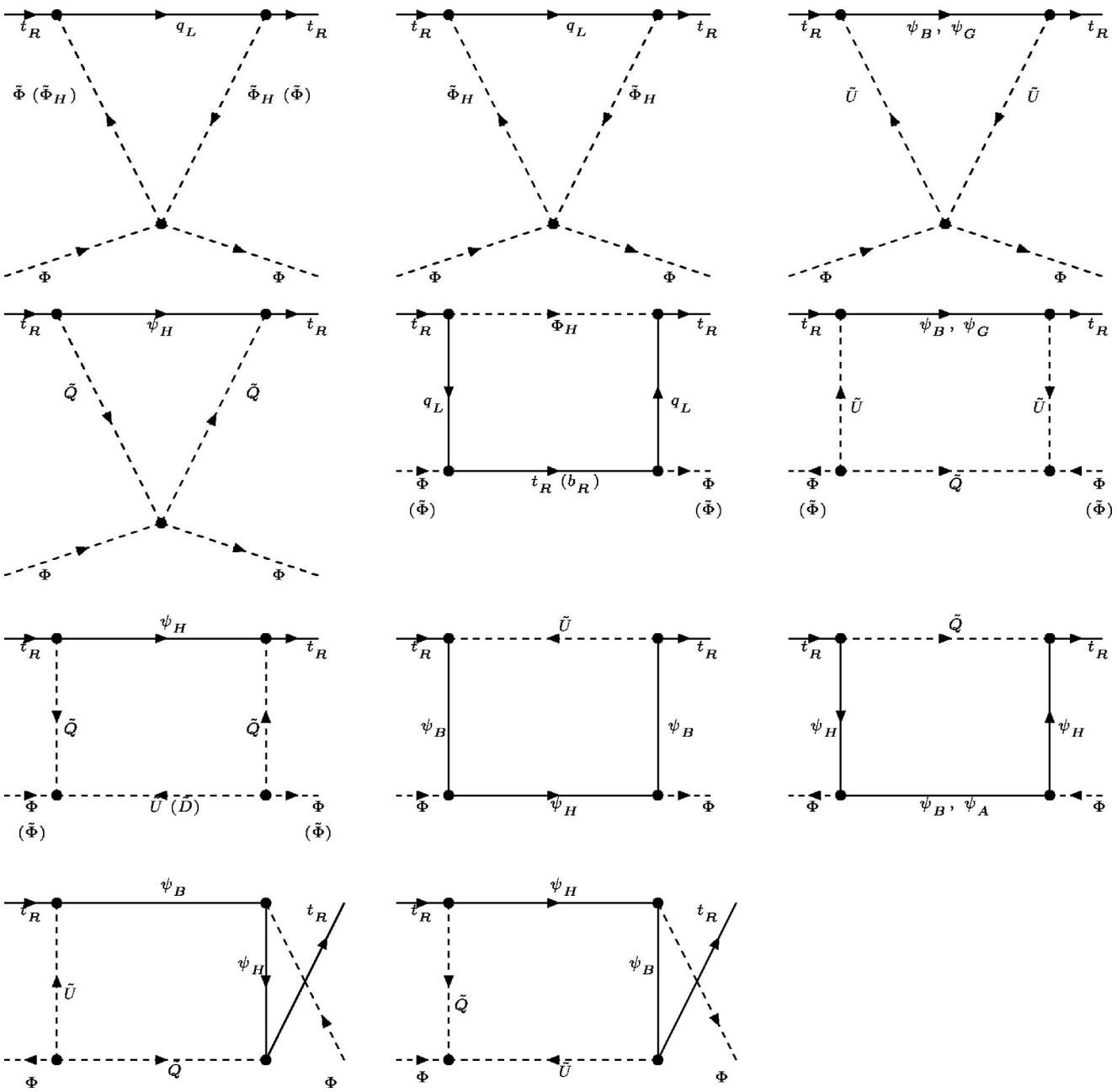

FIG. 5. The Feynman diagrams that induce nontrivial contributions to the Wilson coefficients of the operators $\mathcal{O}_{t\Phi i}$ ($i=1,2,3$).

$$\Sigma_{ij}^{L,0}=[\delta_{ij}+A_{ij}^{L}+B_{ij}^{L}p^2]\not{p},$$

$$\Sigma_{ij}^{R,0}=[\delta_{ij}+A_{ij}^{R}+B_{ij}^{R}p^2]\not{p}, \tag{20}$$

where the first term δ_{ij} represents the Born approximation part and $A^{L,R}, B^{L,R}$ originate from radiative corrections. From Eq. (19) and Eq. (20) one finds the following form for the renormalized self-energies:

$$\hat{\Sigma}_{ij}^{L}=\left[\delta_{ij}+\frac{1}{2}(\delta Z_{L,ij}^{\dagger}+\delta Z_{L,ij})+A_{ij}^{L}+B_{ij}^{L}p^2\right]\not{p},$$

$$\hat{\Sigma}_{ij}^{R}=\left[\delta_{ij}+\frac{1}{2}(\delta Z_{R,ij}^{\dagger}+\delta Z_{R,ij})+A_{ij}^{R}+B_{ij}^{R}p^2\right]\not{p}. \tag{21}$$

The explicit expressions for the renormalization constant $\delta Z_{L,R}$ depend upon the renormalization scheme, i.e., the renormalization conditions. Instead of the often used renor-

malization schemes, i.e., the minimal subtraction scheme or the modified minimal subtraction scheme, we adopt here the physical renormalization conditions

$$\hat{\Sigma}^L_{ij} f_{L,j}|_{\not p=0}=0,$$

$$\hat{\Sigma}^R_{ij} f_{R,j}|_{\not p=0}=0,$$

$$\frac{1}{\not p}\hat{\Sigma}^L_{ij} f_{L,j}|_{\not p=0}=f_{L,i},$$

$$\frac{1}{\not p}\hat{\Sigma}^R_{ij} f_{R,j}|_{\not p=0}=f_{R,i}. \tag{22}$$

The first two conditions mean that the renormalized fields satisfy the equations of motion of free particles (for massless fermions this is a trivial constraint), and the last two conditions set the residue of the propagators at the pole equal to unity. In fact, this scheme is just the on-shell renormalization scheme often used when calculating radiative corrections to electroweak processes [23]. Of course, for high energy processes we can ignore the fermion mass in our approximation. Using the condition Eq. (22), we achieve the renormalized fermion self-energies

$$\hat{\Sigma}^L_{ij}=B^L_{ij}p^2\not p,$$

$$\hat{\Sigma}^R_{ij}=B^R_{ij}p^2\not p. \tag{23}$$

We can attribute these terms to the contributions of the high-dimension operators $\overline{q_L}(i\not D)^3 q_L$, $\overline{t_R}(i\not D)^3 t_R$. After the matching of the full and effective theories, there is no contribution to the operators of our interest given in Eq. (12) and Eq. (13) from the fermion self-energy diagrams.

For the Higgs sector, the bare self-energies are given as

$$\Sigma^0_{\Phi\Phi}(p^2)=D_{\Phi\Phi}+(1+E_{\Phi\Phi})p^2+F_{\Phi\Phi}p^4,$$

$$\Sigma^0_{\Phi\Phi_H}(p^2)=D_{\Phi\Phi_H}+E_{\Phi\Phi_H}p^2+F_{\Phi\Phi_H}p^4, \tag{24}$$

where p denotes the momentum of the external particle. In Eq. (24), D,E,F are standard integral functions that appear in radiative corrections. For the renormalization of the Higgs boson wave function and mass, we require the renormalized boson self-energy to satisfy the conditions

$$\hat{\Sigma}_{\Phi\Phi}(p^2)\big|_{p^2=0}=0,$$

$$\frac{1}{p^2}\hat{\Sigma}_{\Phi\Phi}(p^2)\big|_{p^2=0}=0,$$

$$\hat{\Sigma}_{\Phi\Phi_H}(p^2)\big|_{p^2=0}=0,$$

$$\hat{\Sigma}_{\Phi\Phi_H}(p^2)\big|_{p^2=m_H^2}=0. \tag{25}$$

It is easy to find the renormalized Higgs field self-energies which meet the conditions of Eq. (25):

$$\hat{\Sigma}_{\Phi\Phi}(p^2)=F_{\Phi\Phi}p^4,$$

$$\hat{\Sigma}_{\Phi\Phi_H}(p^2)=F_{\Phi\Phi_H}p^2(p^2-m_H^2). \tag{26}$$

The function $F_{\Phi\Phi}$ is attributed to the contribution of the high-dimensional operator $\Phi^\dagger(D_\mu D^\mu)^2\Phi$. After the matching procedure, this piece will not contribute to the operators in Eqs. (12) and (13) that we are interested in. In fact, after the matching the only nonvanishing contributions from the self-energy diagrams to these operators originate from the integral function $F_{\Phi\Phi_H}$, because we integrate the heavy Higgs doublet out in the effective theory.

After these preparations, we can now derive the Wilson coefficients of the operators $\mathcal{O}_{tq\Phi i}$ $(i=2,3,4,5)$ and $\mathcal{O}_{tq\Phi i}$ $(i=7,8,9,10)$. For clarity, we present their lengthy expressions in Appendix A.

B. The anomalous couplings $\mathcal{O}_{t\Phi}$

This class of anomalous couplings includes the effective operators

$$\mathcal{O}_{t\Phi 1}=i(\Phi^\dagger D_\mu\Phi-(D_\mu\Phi)^\dagger\Phi)\overline{t_R}\gamma_\mu t_R,$$

$$\mathcal{O}_{t\Phi 2}=i(\Phi^\dagger\Phi)(\overline{t_R}\gamma^\mu(D_\mu t_R)-(\overline{D_\mu t_R})\gamma^\mu t_R),$$

$$\mathcal{O}_{t\Phi 3}=i(\Phi^\dagger D_\mu\Phi+(D_\mu\Phi)^\dagger\Phi)\overline{t_R}\gamma_\mu t_R, \tag{27}$$

where the operators $\mathcal{O}_{t\Phi 1},\mathcal{O}_{t\Phi 2}$ have even *CP* parity and $\mathcal{O}_{t\Phi 3}$ has odd *CP* parity. In Fig. 5, we present those Feynman diagrams that induce nontrivial contributions to the Wilson coefficients when matching the amplitude obtained in the effective theory with that in the full theory (MSSM). The ensuing Wilson coefficients are collected in Appendix B.

In the full theory, we also include the one-particle irreducible (1PI) diagrams depicted in Fig. 6, where the gray blobs represent the corresponding diagrams of Fig. 2. However, the contributions from these diagrams disappear as a result of the matching of the effective theory and full theory amplitudes. In order to demonstrate this, let us consider an example. From Sec. III A, we find that the contributions of the subdiagram (framed by the dashed lines) in Fig. 7(a) induce the following term in the effective Lagrangian:

$$\mathcal{L}'_{eff}=\frac{1}{2\mu^2_{NP}}\sum_{\alpha=2}^{5}(C'_{tq\Phi\alpha}+C'_{tq\Phi(5+\alpha)})(\mathcal{O}_{tq\Phi\alpha}+\mathcal{O}_{tq\Phi(5+\alpha)}), \tag{28}$$

where

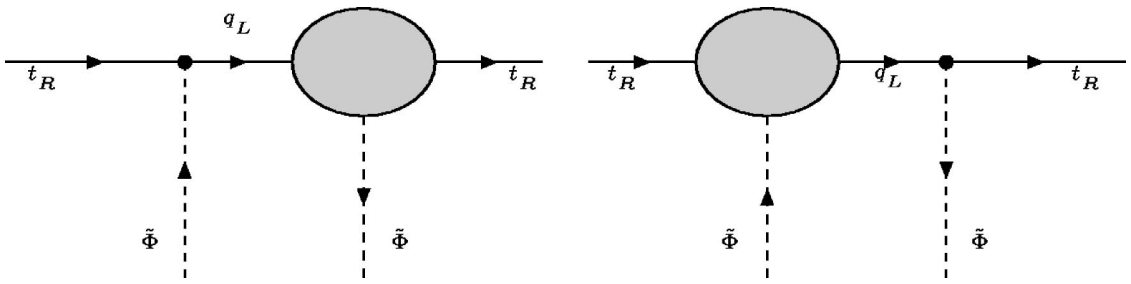

FIG. 6. The one-particle-irreducible Feynman diagrams that are related to the Wilson coefficients of the operators $\mathcal{O}_{t\Phi i}$ $(i=1,2,3)$ in the full theory. The gray bulbs represent the diagrams of Fig. 2.

$$C'_{tq\Phi 2}=-\frac{1}{48\pi^2}g_1^2\sum_I\ [\Lambda^T_{U,I}(\mathcal{C}^1_{121}(x_\mu,x_1,x_{U_I})-2s_\beta x_\mu\mathcal{C}^1_{131}(x_\mu,x_1,x_{U_I}))+2c_\beta\Lambda^{R,1}_{U,I}\mathcal{C}^0_{131}(x_\mu,x_1,x_{U_I})],$$

$$C'_{tq\Phi 3}=\frac{1}{24\pi^2}g_1^2\sum_I\ x_{U_I}[s_\beta\Lambda^T_{U,I}\mathcal{C}^1_{113}(x_\mu,x_1,x_{U_I})-c_\beta\Lambda^{R,1}_{U,I}\mathcal{C}^0_{113}(x_\mu,x_1,x_{U_I})],$$

$$C'_{tq\Phi 4}=-\frac{1}{48\pi^2}g_1^2\sum_I\ [\Lambda^T_{U,I}(Q_9(x_1,x_\mu,x_{U_I})-2s_\beta Q_4(x_\mu,x_1,x_{U_I}))+2c_\beta\Lambda^{R,1}_{U,I}Q_3(x_\mu,x_1,x_{Q_I})],$$

$$C'_{tq\Phi 5}=-\frac{1}{48\pi^2}g_1^2\sum_I\ \Lambda^T_{U,I}\mathcal{C}^1_{112}(x_\mu,x_1,x_{U_I}),$$

$$C'_{tq\Phi 7}=-\frac{i}{48\pi^2}g_1^2\sum_I\ [\Lambda^C_{U,I}(\mathcal{C}^1_{121}(x_\mu,x_1,x_{U_I})-2s_\beta x_\mu\mathcal{C}^1_{131}(x_\mu,x_1,x_{U_I}))+2c_\beta\Lambda^{A,1}_{U,I}\mathcal{C}^0_{131}(x_\mu,x_1,x_{U_I})],$$

$$C'_{tq\Phi 8}=\frac{i}{24\pi^2}g_1^2\sum_I\ x_{U_I}[s_\beta\Lambda^C_{U,I}\mathcal{C}^1_{113}(x_\mu,x_1,x_{U_I})-c_\beta\Lambda^{A,1}_{U,I}\mathcal{C}^0_{113}(x_\mu,x_1,x_{U_I})],$$

$$C'_{tq\Phi 9}=\frac{i}{48\pi^2}g_1^2\sum_I\ [\Lambda^C_{U,I}(Q_9(x_1,x_\mu,x_{U_I})-2s_\beta Q_4(x_\mu,x_1,x_{UI}))+2c_\beta\Lambda^{A,1}_{U,I}Q_3(x_\mu,x_1,x_{Q_I})],$$

$$C'_{tq\Phi 10}=-\frac{i}{48\pi^2}g_1^2\sum_I\ \Lambda^C_{U,I}\mathcal{C}^1_{112}(x_\mu,x_1,x_{U_I}), \tag{29}$$

and $x_\mu=|\mu|^2/\mu^2_{NP}$, $x_{U_I}=m^2_{U_I}/\mu^2_{NP}$ and $x_i=|m_i|^2/\mu^2_{NP}$ $(i=1,2,3)$. The definition of the coupling constants $\Lambda_{U,I}$ and functions $Q_i(x,y,z)$ can be found in Appendix D.

In the effective theory, the amplitude of Fig. 7(b) is written as

$$A^{ET}(p_1,q_1,q_2)=\frac{i}{2}s_\beta\mathbf{h}^{3K}_U(\Phi^\dagger\Phi)\Big\{(C'_{tq\Phi 2}+C'_{tq\Phi 7})\times\bar{t}_R\frac{1}{\not p_1+\not q_1}q_1\cdot p_1 t_R+(C'_{tq\Phi 3}+C'_{tq\Phi 8})\times\bar{t}_R\frac{1}{\not p_1+\not q_1}p_1\cdot(q_1+p_1)t_R+(C'_{tq\Phi 4}+C'_{tq\Phi 9})\bar{t}_R\frac{1}{\not p_1+\not q_1}q_1\cdot(q_1+p_1)t_R-\frac{1}{2}(C'_{tq\Phi 5}+C'_{tq\Phi 10})\times\bar{t}_R\frac{1}{\not p_1+\not q_1}[\not q_1,\not p_1]t_R\Big\}, \tag{30}$$

where p_1,q_1,q_2 denote the four-momenta of the initial right-handed top quark and the Higgs bosons, respectively. In the full theory, the corresponding amplitude takes the form

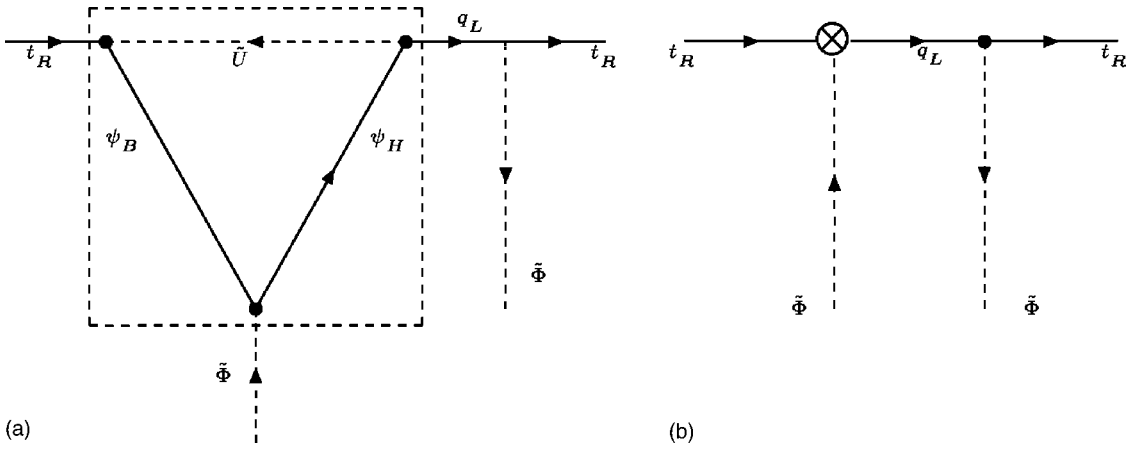

FIG. 7. The 1PI Feynman diagrams related to the Wilson coefficients of the operators $\mathcal{O}_{t\Phi i}$ $(i=1,2,3)$ in (a) the full theory, (b) the effective theory.

$$
\begin{aligned}
A^{FT}(p_1,q_1,q_2) = & -\frac{i}{96\pi^2\mu_{NP}^2} s_\beta g_1^2 (\mathbf{h}_U \mathbf{h}_U^\dagger \mathcal{Z}_U^\dagger)^{3I} \mathcal{Z}_U^{I3} (\Phi^\dagger\Phi) \Big\{ 4[c_\beta \mu_{NP}^2 \sqrt{x_\mu x_1} e^{i(\varphi_1+\varphi_\mu)} \mathcal{C}^0_{111}(x_\mu,x_1,x_{U_I}) \\
& - s_\beta \mathcal{C}^1_{111}(x_\mu,x_1,x_{U_I})] \overline{t}_R \frac{1}{\not p_1 + \not q_1} t_R + [2\mathcal{C}^1_{121}(x_\mu,x_1,x_{U_I}) + 2\mathcal{C}^1_{112}(x_\mu,x_1,x_{U_I}) \\
& + 4c_\beta \sqrt{x_\mu x_1} e^{i(\varphi_1+\varphi_\mu)} Q_3(x_\mu,x_1,x_{U_I}) - 4 s_\beta Q_4(x_\mu,x_1,x_{U_I})] \overline{t}_R \frac{1}{\not p_1 + \not q_1} q_1\cdot(q_1+p_1) t_R - [2\mathcal{C}^1_{121}(x_\mu,x_1,x_{U_I}) \\
& + 4c_\beta x_\mu \sqrt{x_\mu x_1} e^{i(\varphi_1+\varphi_\mu)} \mathcal{C}^0_{131}(x_\mu,x_1,x_{U_I}) - 4 s_\beta x_\mu \mathcal{C}^1_{131}(x_\mu,x_1,x_{U_I})] \overline{t}_R \frac{1}{\not p_1 + \not q_1} q_1\cdot p_1 t_R \\
& + 4x_{U_I}[c_\beta x_\mu \sqrt{x_\mu x_1} e^{i(\varphi_1+\varphi_\mu)} \mathcal{C}^0_{113}(x_\mu,x_1,x_{U_I}) - s_\beta \mathcal{C}^1_{113}(x_\mu,x_1,x_{U_I})] \overline{t}_R \frac{1}{\not p_1 + \not q_1} p_1\cdot(q_1+p_1) t_R \\
& - \mathcal{C}^1_{112}(x_\mu,x_1,x_{U_I}) \overline{t}_R \frac{1}{\not p_1 + \not q_1} [\not q_1, \not p_1] t_R \Big\},
\end{aligned} \tag{31}
$$

where φ_μ and φ_i $(i=1,2,3)$ denote the *CP* phases of the parameter μ and m_i, respectively. As already mentioned, the first term of Eq. (31) is related to the Yukawa coupling renormalization in the full theory and it does not affect our computation. While matching Eq. (31) with Eq. (30), we find that the diagram does not contribute to the Wilson coefficients of the operators $\mathcal{O}_{t\Phi}$. A similar conclusion is true also for the other 1PI diagrams in Fig. 6.

C. The anomalous couplings $\mathcal{O}_{q\Phi}$

This class of anomalous couplings includes the effective operators

$$
\begin{aligned}
\mathcal{O}_{q\Phi 1} &= i(\Phi^\dagger D_\mu \Phi - (D_\mu\Phi)^\dagger \Phi) \overline{q}_L \gamma^\mu q_L, \\
\mathcal{O}_{q\Phi 2} &= i(\Phi^\dagger \sigma^a D_\mu \Phi - (D_\mu\Phi)^\dagger \sigma^a \Phi) \overline{q}_L \sigma^a \gamma^\mu q_L, \\
\mathcal{O}_{q\Phi 3} &= i(\Phi^\dagger \Phi)(\overline{q}_L \gamma^\mu (D_\mu q_L) - \overline{(D_\mu q_L)} \gamma^\mu qL), \\
\mathcal{O}_{q\Phi 4} &= i(\Phi^\dagger \sigma^a \Phi)(\overline{q}_L \sigma^a \gamma^\mu (D_\mu q_L) - \overline{(D_\mu q_L)} \sigma^a \gamma^\mu q_L), \\
\mathcal{O}_{q\Phi 5} &= i(\Phi^\dagger D_\mu \Phi + (D_\mu\Phi)^\dagger \Phi) \overline{q}_L \gamma^\mu q_L, \\
\mathcal{O}_{q\Phi 6} &= i(\Phi^\dagger \sigma^a D_\mu \Phi + (D_\mu\Phi)^\dagger \sigma^a \Phi) \overline{q}_L \sigma^a \gamma^\mu q_L,
\end{aligned} \tag{32}
$$

where the last two operators are *CP* odd and the others are *CP* even. The Feynman diagrams that induce nontrivial contributions to the Wilson coefficients are presented in Fig. 8. We collect the expressions for the Wilson coefficients of the corresponding operators in Appendix C.

IV. EXPERIMENTAL BOUNDS ON THE WILSON COEFFICIENTS

At present, the most rigorous constraint on the Wilson coefficients considered in this work comes from the decay

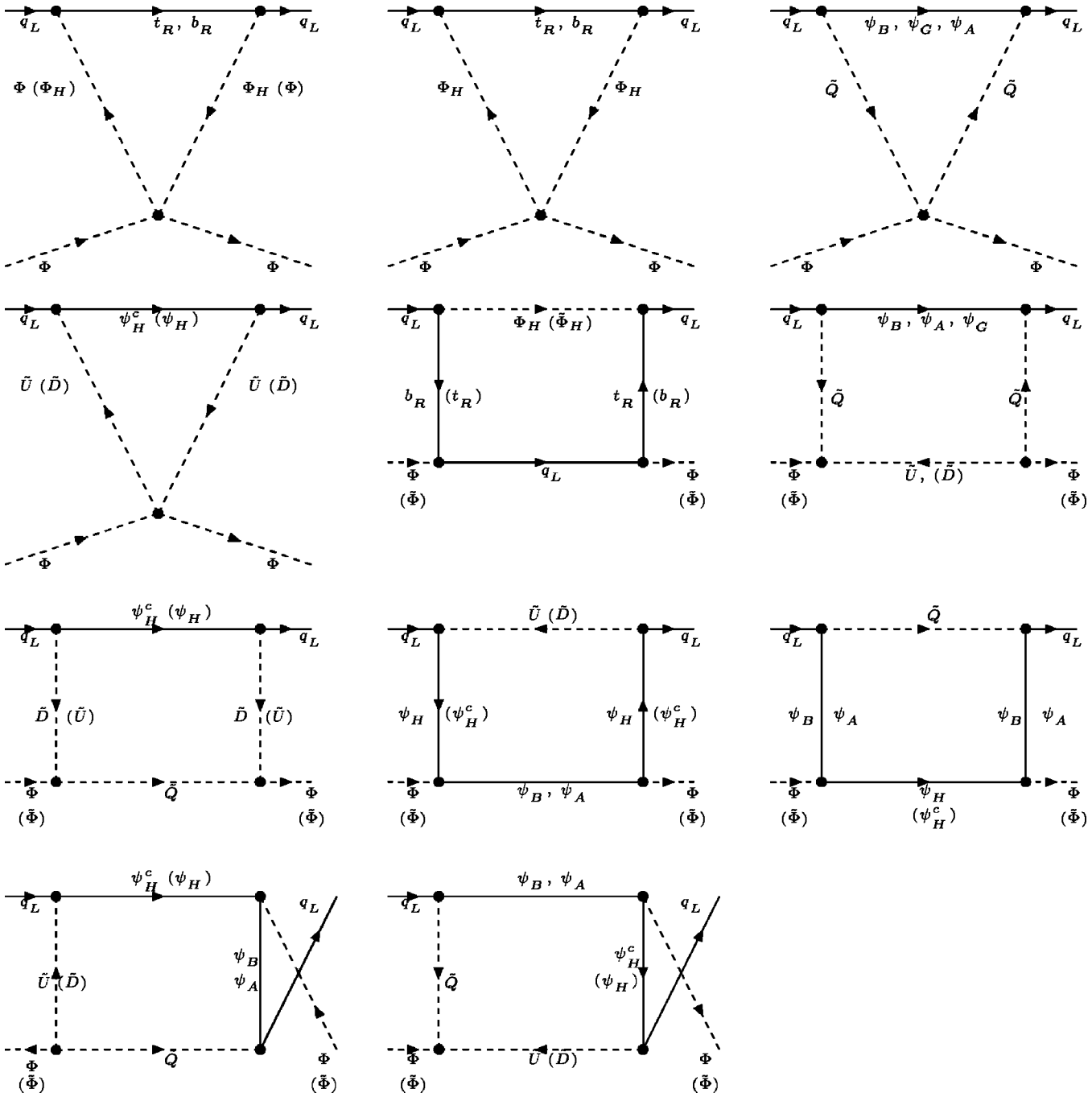

FIG. 8. The Feynman diagrams that induce nontrivial contributions to the Wilson coefficients of the operators $\mathcal{O}_{q\Phi i}$ $(i=1,\ldots,6)$.

$Z\to b\bar{b}$. For an on-shell Z, one can write the general effective vertex $Zb\bar{b}$ as [19]

$$\Gamma_\mu^{Zbb}=-i\frac{e}{4s_W c_W}\left[V_b^Z\gamma_\mu-A_b^Z\gamma_\mu\gamma_5+\frac{1}{2m_b}S_b^Z(p_b-p_{\bar{b}})\right], \tag{33}$$

where $s_W\equiv\sin\theta_W$, $c_W\equiv\cos\theta_W$, and $p_b, p_{\bar{b}}$ are the momenta of the outgoing quark and antiquark, respectively. For the operators listed in Eqs. (12), (13), (27), and (32), $S_b^Z=0$. The vector and axial-vector couplings can be written as

$$V_b^Z=V_b^{Z,0}+\delta V_b^Z,$$
$$A_b^Z=A_b^{Z,0}+\delta A_b^Z, \tag{34}$$

where $V_b^{Z,0}, A_b^{Z,0}$ represent the SM couplings and $\delta V_b^Z, \delta A_b^Z$ are the new physics contributions. Ignoring the bottom quark mass, the lowest-order theoretical prediction on the observable R_b at the Z pole is given by

$$R_b = \frac{\Gamma(Z \to b\bar{b})}{\Gamma(Z \to \text{hadrons})} = R_b^{SM}\left\{1 + 2\frac{V_b^{Z,0}\delta V_b^Z + A_b^{Z,0}\delta A_b^Z}{(V_b^{Z,0})^2 + (A_b^{Z,0})^2}(1 - R_b^{SM})\right\}. \quad (35)$$

With the Born approximation, we can obtain modifications to the couplings V_b^Z, A_b^Z induced by the new physics operators $\mathcal{O}_{q\Phi 1}$ and $\mathcal{O}_{q\Phi 2}$. Provided that there is no accidental cancellation between these contributions, the corrections are given by [20]

$$\delta V_b^Z = \delta A_b^Z = \frac{2 s_W m_W v}{e \mu_{NP}^2}[C_{q\Phi 1} + C_{q\Phi 2}], \quad (36)$$

where v denotes the VEV of the SM Higgs field doublet and m_W is the W-boson mass. From Eq. (35), we have

$$\delta V_b^Z = \delta A_b^Z = \frac{R_b^{exp} - R_b^{SM}}{(1 - R_b^{SM}) R_b^{SM}} \frac{(V_b^{Z,0})^2 + (A_b^{Z,0})^2}{2(V_b^{Z,0} + A_b^{Z,0})}. \quad (37)$$

The SM prediction on R_b and the most recent experimental value are, respectively, given by [24]

$$R_b^{SM} = 0.21572 \pm 0.00015, \quad R_b^{expt} = 0.21664 \pm 0.00065. \quad (38)$$

If we attribute the difference of these two values to the new physics effects, we get a bound for the new physics corrections on the R_b. At the 1σ tolerance we obtain

$$0.00012 \leqslant \Delta R_b \leqslant 0.00172. \quad (39)$$

Correspondingly, the bound for the Wilson coefficients is

$$3.1 \times 10^{-4} \leqslant \frac{v^2}{\mu_{NP}^2} C_{q\Phi(1+2)} \leqslant 4.5 \times 10^{-3} \quad (40)$$

with $C_{q\Phi(1+2)} = C_{q\Phi 1} + C_{q\Phi 2}$. Using the same method, we can also analyze the forward-backward asymmetry A_{FB}^b of the decay $Z \to b\bar{b}$. However, our theoretical result indicates that the present experimental data on this quantity set a weaker bound on the Wilson coefficients than on R_b.

The other Wilson coefficients of the operators appearing in the Lagrangian are not constrained by R_b on the Born approximation level. With higher-order approximations, those operators contribute to the gauge boson self-energies, and thus we can get for them only a rather loose bound with a significant uncertainty. We can also have loose bounds from the argument of partial wave unitarity [25]:

$$|C_{tq\Phi 1}| \leqslant \frac{16\pi}{3\sqrt{2}}\left(\frac{\mu_{NP}}{v}\right), \quad |C_{t\Phi 1}| \leqslant 8\pi\sqrt{3},$$
$$-6.4 \leqslant C_{tq\Phi 2} \leqslant 10.4. \quad (41)$$

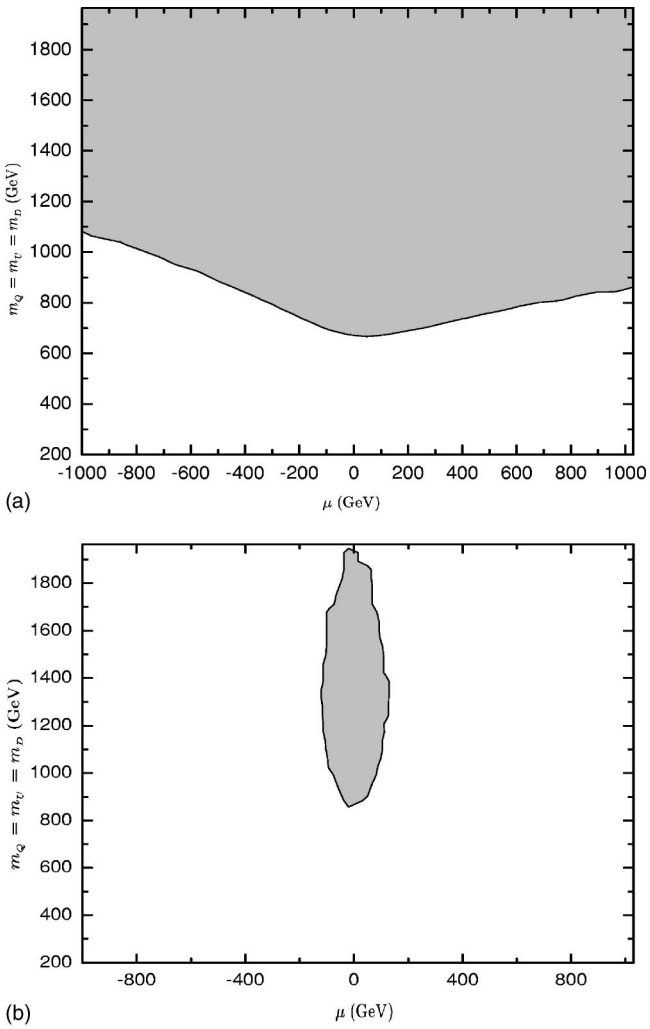

FIG. 9. The constraint from the anomalous coupling $\mathcal{O}_{q\Phi 1,2}$ set by the R_b experimental data with 1σ tolerance on the soft breaking parameters $m_Q = m_U = m_D$ versus the μ parameter in the superpotential with $m_1 = m_2 = m_3 = 500$ GeV, $A_t = A_b = 100$ GeV, $m_H = 500$ GeV, and (a) $\tan\beta = 2$, (b) $\tan\beta = 40$.

At present, there are no strong experimental constraints on the *CP*-odd couplings involving the top quark.

It is well known that the MSSM contains in its general form unfortunately many "new" free parameters in addition to the SM parameters. In order to simplify our discussion, we take the following assumptions to restrict the MSSM parameter space.

(1) All possible *CP* phases are taken to be zero or π. A direct consequence of this choice is that there are no *CP*-odd operators in the effective Lagrangian Eq. (1).

(2) All Yukawa couplings and the soft breaking parameters are flavor conserving, i.e., the mixing matrices $\mathcal{Z}_Q = \mathcal{Z}_U = \mathcal{Z}_D = \mathbf{1}$.

Under these assumptions, the parameters relevant to our discussion are the gauge coupling constants g_1, g_2, g_3, the Higgsino and gaugino masses μ, m_1, m_2, m_3, the Yukawa couplings of the third generation quarks and the corresponding soft breaking parameters $h_t = h_U^{33}, h_b = h_D^{33}, A_t = A_U^{33}, A_b = A_D^{33}$, and the square masses of the heavy Higgs boson

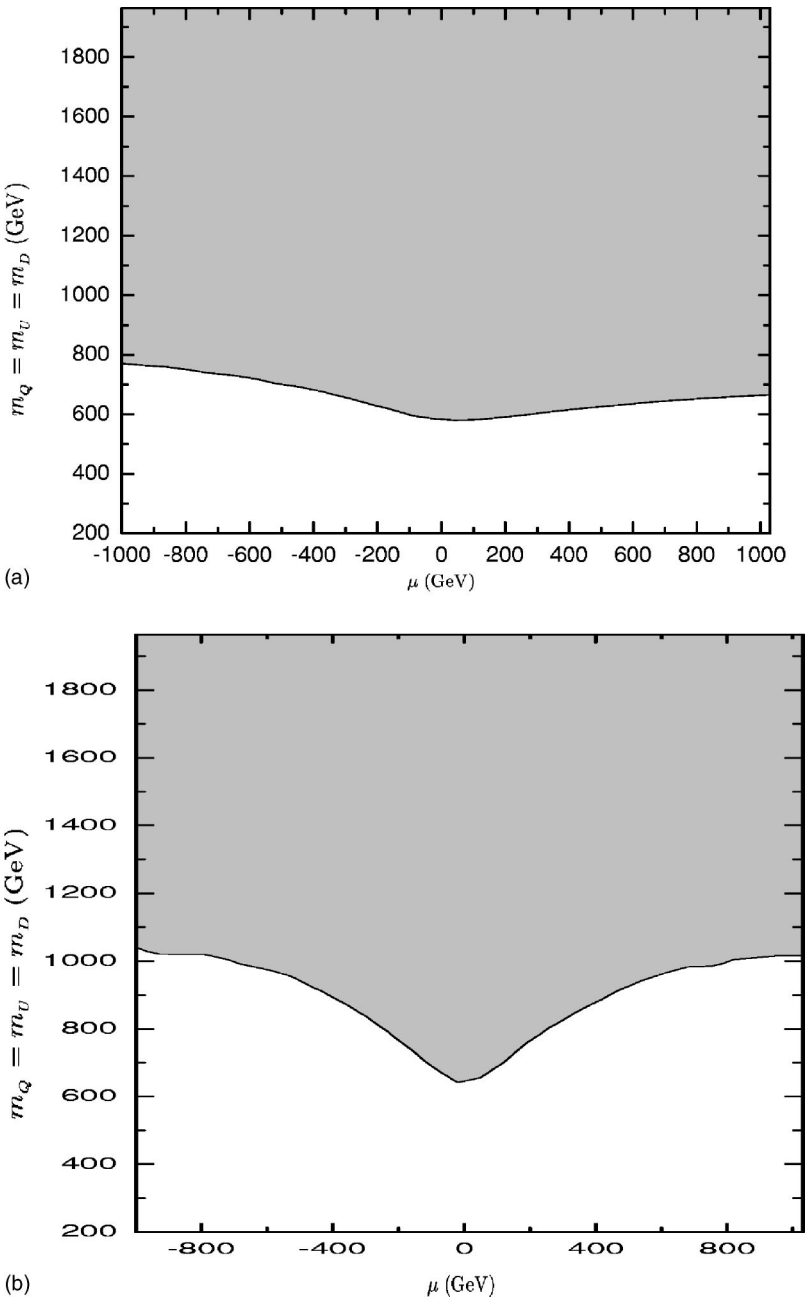

FIG. 10. Relaxing the lower bound to -5×10^{-5} and keeping the upper bound unchanged as in Eq. (40), the constraint from the anomalous coupling $\mathcal{O}_{q\Phi1,2}$ on the soft breaking parameters $m_Q=m_U=m_D$ versus the μ parameter in the superpotential with $m_1=m_2=m_3=500$ GeV, $A_t=A_b=100$ GeV, $m_H=500$ GeV and (a) $\tan\beta=2$, (b) $\tan\beta=40$.

doublet and the third generation squarks $m_H^2, m_{Q_I}^2, m_{U_I}^2, m_{D_I}^2$ $(I=3)$. In our numerical analysis, we will disregard the loose bounds from partial wave unitarity on the Wilson coefficients $C_{tq\Phi1}, C_{t\Phi1}, C_{tq\Phi2}$ due to the large uncertainties mentioned above.

Without losing generality, we assume $m_Q=m_U=m_D$, $A_t=A_b$, $m_1=m_2=m_3$ in our numerical computations. Setting $\mu_{NP}=1000$ GeV, $m_H=500$ GeV, $m_1=m_2=m_3=500$ GeV, $A_t=A_b=100$ GeV, we obtain the constraints set by Eq. (40) on the soft breaking parameters. In Fig. 9, we plot the values of $m_Q=m_U=m_D$ versus the parameter μ with (a) $\tan\beta=2$, and (b) $\tan\beta=40$, where the gray regions are allowed by the condition for $v^2C_{q\Phi(1+2)}/\mu_{NP}^2$ set by Eq. (40). From this plot we observe that the restriction set on the parameter space with $\tan\beta=40$ is more rigorous than that with $\tan\beta=2$. At $\tan\beta=2$, the contribution from the supersymmetric box diagrams varies from negative to positive gradually and then tends to zero after its maximum as $m_Q=m_U=m_D$ increase from 200 GeV. When $m_Q=m_U=m_D\geqslant1.3$ TeV, the contribution is definitely less than 10^{-4}. In addition to those box diagrams, $C_{q\Phi1}$ also receives a contribution from the heavy Higgs doublet. Under our choice about the parameter space, the Higgs contribution to the Wilson coefficient $v^2C_{q\Phi1}/\mu_{NP}^2$ is proportional to $[1/\tan^2\beta-(m_b/m_t)^4\tan^2\beta]$. Taking the bottom quark mass $m_b=4.5$ GeV and top quark mass $m_t=174$ GeV, this contribution is about 5×10^{-4} for $\tan\beta=2$. As $\tan\beta$ increases, the contribution of the heavy Higgs doublet is strongly suppressed and is less than 10^{-7} for $\tan\beta=40$. This fact can help us to understand why very massive supersymmetry par-

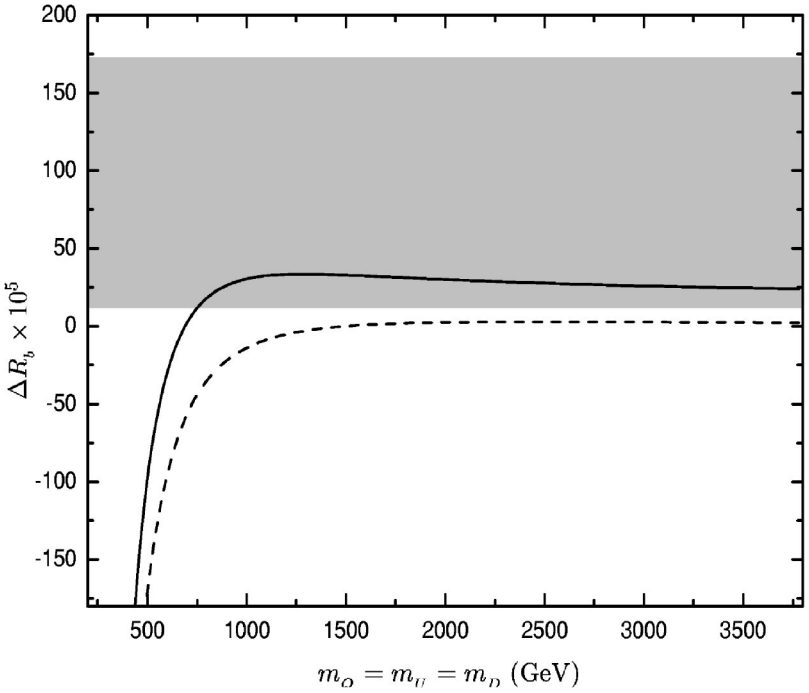

FIG. 11. Taking $\mu = m_i = 500$ GeV ($i = 1,2,3$), $A_t = A_b = 100$ GeV, $m_H = 500$ GeV, ΔR_b versus squark masses $m_Q = m_U = m_D$ with $\tan\beta = 2$ (solid line) or $\tan\beta = 40$ (dashed line).

ticles are allowed by the experimental bound for $\tan\beta = 2$ [Fig. 9(a)], whereas most of the parameter space is excluded by the bound except a narrow band in the neighborhood of $\mu = 0$ for $\tan\beta = 40$ [Fig. 9(b)].

In the figures discussed above, we considered a 1σ tolerance for the experimental data. Since the central value of the experimentally measured R_b is only about one standard deviation away from the SM prediction, this sets a lower bound on the $C_{q\Phi(1+2)}$ which is positive and very close to zero as shown in Eq. (40). Certainly, very massive supersymmetry particles are excluded by this condition in the large $\tan\beta$ case. In fact, considering the practical situation of the experiments, we may relax the lower bound on the $C_{q\Phi(1+2)}$ to -5×10^{-5}, while the upper bound remains unchanged (this is only slightly beyond the standard deviation). In Fig. 10 we plot $m_Q = m_U = m_D$ versus μ by using the constraint $-5\times10^{-5} \leqslant v^2 C_{q\Phi(1+2)}/\mu_{NP}^2 \leqslant 4.5\times10^{-3}$. One can see that the allowed parameter region is drastically enlarged in comparison with the case of strict 1σ tolerance for large $\tan\beta$.

Now, we discuss the operator $\mathcal{O}_{q\Phi1,2}$ corrections to R_b in the MSSM. Taking $\mu = m_i = 500$ GeV ($i = 1,2,3$), we plot ΔR_b versus the squark masses $m_Q = m_U = m_D$ with $\tan\beta = 2,40$ in Fig. 11. The gray band is the experimentally allowed region at the 1σ tolerance. When the scalar quark mass is less than 700 GeV, the supersymmetric box diagrams determine the leading contribution and result in a negative ΔR_b; the corresponding parameter space is excluded by Eq. (39). As the parameters $m_Q = m_U = m_D$ increase, the supersymmetric contribution becomes positive and then tends to zero after a maximum. With $\tan\beta = 2$, the correction of the

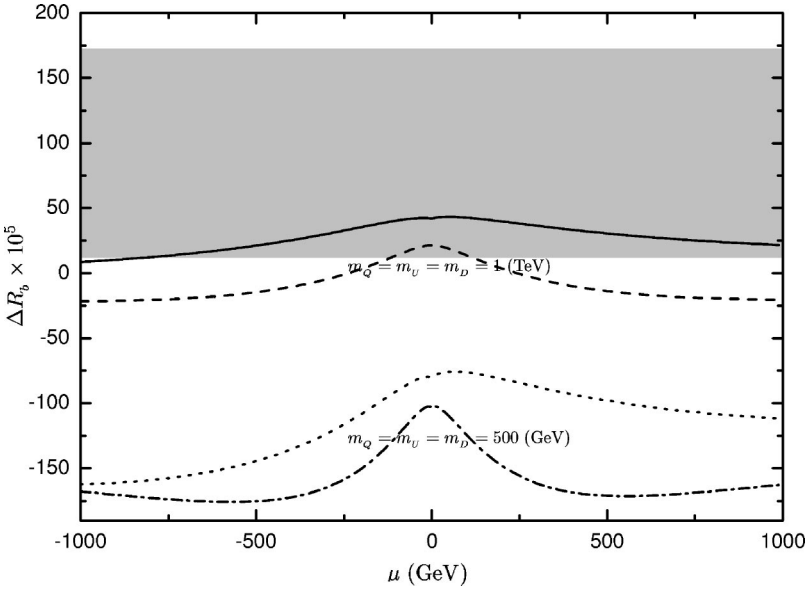

FIG. 12. ΔR_b versus the parameter μ with $m_i = 500$ GeV ($i = 1,2,3$), $A_t = A_b = 100$ GeV, $m_H = 500$ GeV. Solid line, $m_Q = m_U = m_D = 1$ TeV, $\tan\beta = 2$; dashed line, $m_Q = m_U = m_D = 1$ TeV, $\tan\beta = 40$; dotted line, $m_Q = m_U = m_D = 500$ GeV, $\tan\beta = 2$; dot-dashed line, $m_Q = m_U = m_D = 500$ GeV, $\tan\beta = 40$.

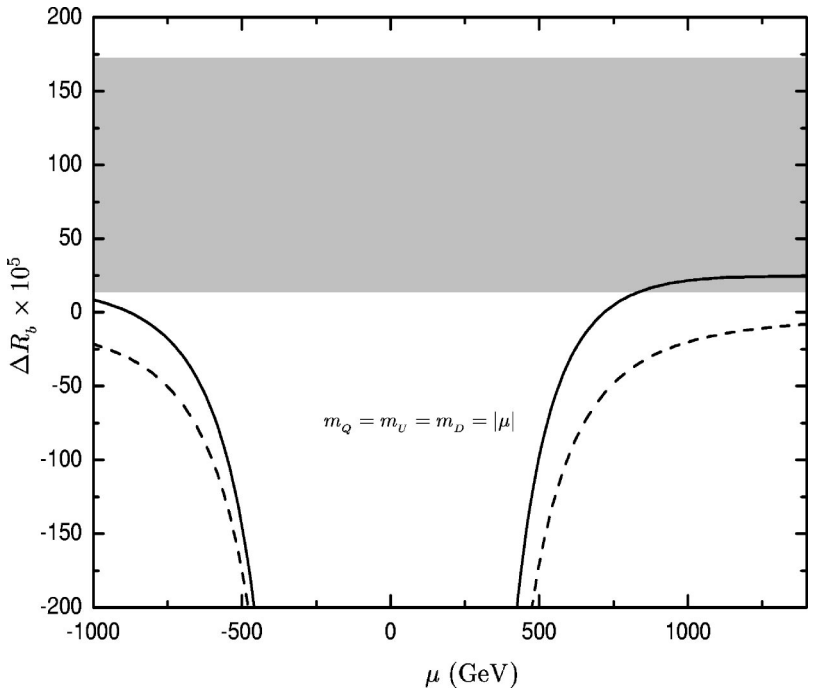

FIG. 13. Under the assumption $m_Q = m_U = m_D = |\mu|$, ΔR_b versus the parameter μ. The other parameters are taken as $m_i = 500$ GeV ($i = 1,2,3$), $A_t = A_b = 100$ GeV, $m_H = 500$ GeV, and $\tan\beta = 2$ (solid line), $\tan\beta = 40$ (dashed line).

heavy Higgs doublet to the ΔR_b is about 2.3×10^{-4} and plays the leading role when $m_Q = m_U = m_D \geqslant 1.3$ TeV. For $\tan\beta = 40$, the total corrections from the Higgs and supersymmetric sectors to R_b do not satisfy Eq. (39), because the contribution of the heavy Higgs doublet is strongly suppressed. For $-\mu = m_i = 500$ GeV ($i = 1,2,3$), the plot is similar to Fig. 11 and not shown here. Taking $m_i = 500$ GeV ($i = 1,2,3$), $m_Q = m_U = m_D = 500,1000$ GeV, and $\tan\beta = 2,40$, we present ΔR_b versus the parameter μ in Fig. 12. For $m_Q = m_U = m_D = 500$ GeV (dotted and dot-dashed lines), the corresponding parameter space is excluded by the condition Eq. (39) due to the negative supersymmetry contribution. With $m_Q = m_U = m_D = 1$ TeV and $\tan\beta = 2$ (solid line), ΔR_b satisfies the condition Eq. (39) when $\mu \geqslant -700$ GeV. As for the case $m_Q = m_U = m_D = 1$ TeV and $\tan\beta = 40$ (dashed line), the new physics correction to ΔR_b is excluded by the 1σ tolerance experimental bound except in the region neighboring $\mu = 0$ GeV. Finally, we investigate the new physics prediction on ΔR_b with the assumption $m_Q = m_U = m_D = |\mu|$. Choosing $m_i = 500$ GeV ($i = 1,2,3$), $\tan\beta = 2,40$, we plot ΔR_b versus the parameter $m_Q = m_U = m_D = |\mu|$ in Fig. 13. For the case $\tan\beta = 40$, the correction to ΔR_b exceeds the 1σ tolerance experimental bound. As $\tan\beta = 2$ and $\mu > 800$ GeV, the new physics prediction on ΔR_b satisfies this bound because of the relatively large contribution from the heavy Higgs boson. In those analyses the experimental bound with 1σ standard deviation is adopted. After we relax the condition [Eq. (39)] slightly, the more massive supersymmetry particles are also permitted by the corresponding experimental bound.

Since the experimental data constrain the coefficients $C_{q\Phi 1,2}$ strongly, the operators $\mathcal{O}_{q\Phi 1,2}$ have only negligible effects on the measurements at proposed future colliders [19]. Other operators will produce the observable effects in the next generation colliders. In the associated production of the Higgs boson and top quark pair $e^+e^- \to t\bar{t}h$, the *CP*-even operators will affect the energy and angular distributions of the final state particles [20]. Through measurements of various distributions, such as $d\sigma/dE_t$, $d\sigma/dE_h$, and $d\sigma/d\cos\theta_h$ (E_t,E_h denote the outgoing energy of the top quark and Higgs boson, respectively; θ_h is the angle of three-momentum of the outgoing Higgs boson with respect to the electron beam direction), we can obtain useful information about the operators. The constraints on the *CP*-odd operators can be obtained through measuring various *CP* violation observables in this process. In the process $e^+e^- \to t\bar{t}$, we can analyze the effects of the operators on various polarized top quark production cross sections. On the other hand, more strict constraints on the supersymmetry parameter space will be set by more precise measurements on the widths of $Z \to b\bar{b}$ and top quark decays. All of these will provide valuable information for the search for supersymmetry particles at future colliders.

It should be stressed that the above numerical analysis is performed under special assumptions about the MSSM parameter space. For example, we assume that all the parameters are real and flavor conserving, the universal soft parameters are $m_Q = m_U = m_D$, $A_t = A_b$, $m_1 = m_2 = m_3$. In a practical phenomenology analysis, those *a priori* conditions should be dismissed. Nevertheless, it is quite obvious that the experimental data on R_b set significant bounds on the parameter space even in a more general case than the one we have considered here.

V. SUMMARY

We have considered in this work the anomalous couplings between top quark and Higgs boson induced by the MSSM

when the heavy Higgs doublet and all supersymmetry fields are integrated out. An essential assumption made here is that there is only one neutral Higgs boson with an electroweak mass; the other Higgs particles are much heavier. We have derived the Wilson coefficients of the relevant higher-dimensional operators in the ensuing effective theory. We have also studied numerically the constraints set by the experimental results for $R_b=\Gamma(Z\to b\bar{b})/\Gamma(Z\to\text{hadrons})$ on the parameters of the MSSM.

ACKNOWLEDGMENTS

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APPENDIX A: THE WILSON COEFFICIENTS FOR THE OPERATORS $\mathcal{O}_{tq\Phi i}$ ($i=2,\ldots,5,7,\ldots,10$)

$$
\begin{aligned}
C_{tq\Phi 2}=&-\frac{c_\beta}{(4\pi)^2}\Bigg\{\sum_{i=1,2}F_i g_i^2\xi_{Hi}^R P_H(x_\mu,x_i)-s_\beta c_\beta\sum_{q=U,D}(-1)^{1/2-T_Z^q}\mathrm{Re}[\mathbf{h}_U^{33}(\mathcal{Z}_q\hat{\mathbf{A}}_q\mathcal{Z}_Q)_{IJ}(\mathcal{Z}_q\hat{\mathbf{A}}'_q\mathcal{Z}_Q)^\dagger_{JI}]P_H(x_{Q_J},x_{q_I})\\
&+s_\beta c_\beta\mathrm{Re}[\mathbf{h}_U^{33}(\mathcal{Z}_R\hat{\mathbf{A}}_E\mathcal{Z}_L)_{IJ}(\mathcal{Z}_R\hat{\mathbf{A}}'_E\mathcal{Z}_L)^\dagger_{JI}]P_H(x_{L_J},x_{R_I})\Bigg\}+\frac{1}{32\pi^2x_H}s_\beta c_\beta^2\mathrm{Re}(\mathbf{h}_D^\dagger\mathbf{h}_D\mathbf{h}_U^\dagger)_{33}(1-\ln x_H)\\
&+\frac{g_3^2}{6\pi^2}s_\beta\Gamma^{R,3}_{U,IJ}x_{U_J}\mathcal{C}^0_{113}(x_3,x_{Q_I},x_{U_J})+\frac{g_1^2}{72\pi^2}s_\beta\Gamma^{R,1}_{U,IJ}x_{U_J}\mathcal{C}^0_{113}(x_1,x_{Q_I},x_{U_J})+\frac{1}{16\pi^2}c_\beta\Gamma^S_{U,IJ}x_{Q_J}\mathcal{C}^0_{113}(x_\mu,x_{D_I},x_{Q_J})\\
&-\frac{1}{192\pi^2}\sum_{i=1,2}F_i^2g_i^2[\Lambda^T_{Q,I}(\mathcal{C}^1_{211}(x_\mu,x_i,x_{Q_I})+2s_\beta x_\mu\mathcal{C}^1_{311}(x_\mu,x_i,x_{Q_I}))+2c_\beta x_\mu\Lambda^{R,i}_{U,I}\mathcal{C}^0_{311}(x_\mu,x_i,x_{Q_I})]\\
&-\frac{1}{48\pi^2}g_1^2[\Lambda^T_{U,I}(\mathcal{C}^1_{121}(x_\mu,x_1,x_{U_I})+2s_\beta x_\mu\mathcal{C}^1_{131}(x_\mu,x_1,x_{U_I}))+2c_\beta x_\mu\Lambda^{R,1}_{U,I}\mathcal{C}^0_{131}(x_\mu,x_1,x_{U_I})],
\end{aligned}
$$

$$
\begin{aligned}
C_{tq\Phi 3}=&-\frac{1}{32\pi^2x_H}s_\beta c_\beta^2\mathrm{Re}(\mathbf{h}_D^\dagger\mathbf{h}_D\mathbf{h}_U^\dagger)_{33}+\frac{g_3^2}{6\pi^2}s_\beta\Gamma^{R,3}_{U,IJ}Q_1(x_3,x_{Q_I},x_{U_J})+\frac{g_1^2}{72\pi^2}s_\beta\Gamma^{R,1}_{U,IJ}Q_1(x_1,x_{Q_I},x_{U_J})\\
&+\frac{1}{16\pi^2}c_\beta\Gamma^S_{U,IJ}Q_1(x_\mu,x_{D_I},x_{Q_J})-\frac{1}{96\pi^2}\sum_{i=1,2}F_i^2g_i^2x_{Q_I}[s_\beta\Lambda^T_{Q,I}\mathcal{C}^1_{113}(x_\mu,x_i,x_{Q_I})+c_\beta x_\mu\Lambda^{R,i}_{U,I}\mathcal{C}^0_{113}(x_\mu,x_i,x_{Q_I})]\\
&-\frac{1}{24\pi^2}g_1^2x_{U_I}[s_\beta\Lambda^T_{U,I}\mathcal{C}^1_{113}(x_\mu,x_1,x_{U_I})+c_\beta x_\mu\Lambda^{R,1}_{U,I}\mathcal{C}^0_{113}(x_\mu,x_1,x_{U_I})],
\end{aligned}
$$

$$
\begin{aligned}
C_{tq\Phi 4}=&-\frac{c_\beta}{(4\pi)^2}\Bigg\{\sum_{i=1,2}F_i g_i^2\xi_{Hi}^R P_H(x_\mu,x_i)-s_\beta c_\beta\sum_{q=U,D}(-1)^{1/2-T_Z^q}\mathrm{Re}[\mathbf{h}_U^{33}(\mathcal{Z}_q\hat{\mathbf{A}}_q\mathcal{Z}_Q)_{IJ}(\mathcal{Z}_q\hat{\mathbf{A}}'_q\mathcal{Z}_Q)^\dagger_{JI}]P_H(x_{Q_J},x_{q_I})\\
&+s_\beta c_\beta\mathrm{Re}[\mathbf{h}_U^{33}(\mathcal{Z}_R\hat{\mathbf{A}}_E\mathcal{Z}_L)_{IJ}(\mathcal{Z}_R\hat{\mathbf{A}}'_E\mathcal{Z}_L)^\dagger_{JI}]P_H(x_{L_J},x_{R_I})\Bigg\}+\frac{1}{32\pi^2x_H}s_\beta c_\beta^2\mathrm{Re}(\mathbf{h}_D^\dagger\mathbf{h}_D\mathbf{h}_U^\dagger)_{33}(1-\ln x_H)\\
&+\frac{g_3^2}{6\pi^2}s_\beta\Gamma^{R,3}_{U,IJ}x_{Q_I}\mathcal{C}^0_{131}(x_3,x_{Q_I},x_{U_J})+\frac{g_1^2}{72\pi^2}s_\beta\Gamma^{R,1}_{U,IJ}x_{Q_I}\mathcal{C}^0_{131}(x_1,x_{Q_I},x_{U_J})+\frac{1}{16\pi^2}c_\beta\Gamma^S_{U,IJ}x_{D_I}\mathcal{C}^0_{131}(x_\mu,x_{D_I},x_{Q_J})\\
&-\frac{1}{192\pi^2}\sum_{i=1,2}F_i^2g_i^2[\Lambda^T_{Q,I}(Q_9(x_\mu,x_i,x_{Q_I})+2s_\beta Q_2(x_i,x_\mu,x_{Q_I}))+2c_\beta\Lambda^{R,i}_{U,I}Q_1(x_i,x_\mu,x_{Q_I})]\\
&-\frac{1}{48\pi^2}g_1^2[\Lambda^T_{U,I}(Q_9(x_1,x_\mu,x_{U_I})+2s_\beta Q_4(x_\mu,x_1,x_{U_I}))+2c_\beta\Lambda^{R,1}_{U,I}Q_3(x_\mu,x_1,x_{Q_I})],
\end{aligned}
$$

$$C_{tq\Phi 5}=-\frac{1}{32\pi^2 x_H}s_\beta c_\beta^2\mathrm{Re}(\mathbf{h}_D^\dagger\mathbf{h}_D\mathbf{h}_U^\dagger)_{33}-\frac{1}{192\pi^2}\sum_{i=1,2}F_i^2 g_i^2\Lambda^T_{Q,I}\mathcal{C}^1_{112}(x_\mu,x_i,x_{Q_I})-\frac{1}{48\pi^2}g_1^2\Lambda^T_{U,I}\mathcal{C}^1_{112}(x_\mu,x_1,x_{U_I}),$$

$$\begin{aligned}C_{tq\Phi 7}=&\,i\frac{c_\beta}{(4\pi)^2}\Bigg\{\sum_{i=1,2}F_i g_i^2\xi^A_{Hi}P_H(x_\mu,x_i)-s_\beta c_\beta\sum_{q=U,D}(-1)^{1/2-T^q_Z}\mathrm{Im}[\mathbf{h}_U^{33}(\mathcal{Z}_q\hat{\mathbf{A}}_q\mathcal{Z}_Q)_{IJ}(\mathcal{Z}_q\hat{\mathbf{A}}'_q\mathcal{Z}_Q)^\dagger_{JI}]P_H(x_{Q_J},x_{q_I})\\&+s_\beta c_\beta\mathrm{Im}[\mathbf{h}_U^{33}(\mathcal{Z}_R\hat{\mathbf{A}}_E\mathcal{Z}_L)_{IJ}(\mathcal{Z}_R\hat{\mathbf{A}}'_E\mathcal{Z}_L)^\dagger_{JI}]P_H(x_{L_J},x_{R_I})\Bigg\}+\frac{i}{32\pi^2x_H}s_\beta c_\beta^2\mathrm{Im}(\mathbf{h}_D^\dagger\mathbf{h}_D\mathbf{h}_U^\dagger)_{33}(1-\ln x_H)\\&+i\frac{g_3^2}{6\pi^2}s_\beta\Gamma^{A,3}_{U,IJ}x_{U_J}\mathcal{C}^0_{113}(x_3,x_{Q_I},x_{U_J})+i\frac{g_1^2}{72\pi^2}s_\beta\Gamma^{A,1}_{U,IJ}x_{U_J}\mathcal{C}^0_{113}(x_1,x_{Q_I},x_{U_J})+\frac{i}{16\pi^2}c_\beta\Gamma^B_{U,IJ}x_{Q_J}\mathcal{C}^0_{113}(x_\mu,x_{D_I},x_{Q_J})\\&-i\frac{1}{192\pi^2}\sum_{i=1,2}F_i^2g_i^2[\Lambda^C_{Q,I}(\mathcal{C}^1_{211}(x_\mu,x_i,x_{Q_I})+2s_\beta x_\mu\mathcal{C}^1_{311}(x_\mu,x_i,x_{Q_I}))+2c_\beta x_\mu\Lambda^{A,i}_{U,I}\mathcal{C}^0_{311}(x_\mu,x_i,x_{Q_I})]\\&-\frac{i}{48\pi^2}g_1^2[\Lambda^C_{U,I}(\mathcal{C}^1_{121}(x_\mu,x_1,x_{U_I})+2s_\beta x_\mu\mathcal{C}^1_{131}(x_\mu,x_1,x_{U_I}))+2c_\beta x_\mu\Lambda^{A,1}_{U,I}\mathcal{C}^0_{131}(x_\mu,x_1,x_{U_I})],\end{aligned}$$

$$\begin{aligned}C_{tq\Phi 8}=&-\frac{i}{32\pi^2x_H}s_\beta c_\beta^2\mathrm{Im}(\mathbf{h}_D^\dagger\mathbf{h}_D\mathbf{h}_U^\dagger)_{33}+i\frac{g_3^2}{6\pi^2}s_\beta\Gamma^{A,3}_{U,IJ}Q_1(x_3,x_{Q_I},x_{U_J})+i\frac{g_1^2}{72\pi^2}s_\beta\Gamma^{A,1}_{U,IJ}Q_1(x_1,x_{Q_I},x_{U_J})\\&+\frac{i}{16\pi^2}c_\beta\Gamma^B_{U,IJ}Q_1(x_\mu,x_{D_I},x_{Q_J})-\frac{i}{96\pi^2}\sum_{i=1,2}F_i^2g_i^2x_{Q_I}[s_\beta\Lambda^C_{Q,I}\mathcal{C}^1_{113}(x_\mu,x_i,x_{Q_I})+c_\beta x_\mu\Lambda^{A,i}_{U,I}\mathcal{C}^0_{113}(x_\mu,x_i,x_{Q_I})]\\&-\frac{i}{24\pi^2}g_1^2x_{U_I}[s_\beta\Lambda^C_{U,I}\mathcal{C}^1_{113}(x_\mu,x_1,x_{U_I})+c_\beta x_\mu\Lambda^{A,1}_{U,I}\mathcal{C}^0_{113}(x_\mu,x_1,x_{U_I})],\end{aligned}$$

$$\begin{aligned}C_{tq\Phi 9}=&\,i\frac{c_\beta}{(4\pi)^2}\Bigg\{\sum_{i=1,2}F_i g_i^2\xi^A_{Hi}P_H(x_\mu,x_i)-s_\beta c_\beta\sum_{q=U,D}(-1)^{1/2-T^q_Z}\mathrm{Im}[\mathbf{h}_U^{33}(\mathcal{Z}_q\hat{\mathbf{A}}_q\mathcal{Z}_Q)_{IJ}(\mathcal{Z}_q\hat{\mathbf{A}}'_q\mathcal{Z}_Q)^\dagger_{JI}]P_H(x_{Q_J},x_{q_I})\\&+s_\beta c_\beta\mathrm{Im}[\mathbf{h}_U^{33}(\mathcal{Z}_R\hat{\mathbf{A}}_E\mathcal{Z}_L)_{IJ}(\mathcal{Z}_R\hat{\mathbf{A}}'_E\mathcal{Z}_L)^\dagger_{JI}]P_H(x_{L_J},x_{R_I})\Bigg\}+\frac{i}{32\pi^2x_H}s_\beta c_\beta^2\mathrm{Im}(\mathbf{h}_D^\dagger\mathbf{h}_D\mathbf{h}_U^\dagger)_{33}(1-\ln x_H)\\&+i\frac{g_3^2}{6\pi^2}s_\beta\Gamma^{A,3}_{U,IJ}x_{Q_I}\mathcal{C}^0_{131}(x_3,x_{Q_I},x_{U_J})+i\frac{g_1^2}{72\pi^2}s_\beta\Gamma^{A,1}_{U,IJ}x_{Q_I}\mathcal{C}^0_{131}(x_1,x_{Q_I},x_{U_J})+\frac{i}{16\pi^2}c_\beta\Gamma^B_{U,IJ}x_{D_I}\mathcal{C}^0_{131}(x_\mu,x_{D_I},x_{Q_J})\\&-\frac{i}{192\pi^2}\sum_{i=1,2}F_i^2g_i^2[\Lambda^C_{Q,I}(Q_9(x_\mu,x_i,x_{Q_I})+2s_\beta Q_2(x_i,x_\mu,x_{Q_I}))+2c_\beta\Lambda^{A,i}_{U,I}Q_1(x_i,x_\mu,x_{Q_I})]\\&-\frac{i}{48\pi^2}g_1^2[\Lambda^C_{U,I}(Q_9(x_1,x_\mu,x_{U_I})+2s_\beta Q_4(x_\mu,x_1,x_{U_I}))+2c_\beta\Lambda^{A,1}_{U,I}Q_3(x_\mu,x_1,x_{Q_I})],\end{aligned}$$

$$C_{tq\Phi 10}=-\frac{i}{32\pi^2x_H}s_\beta c_\beta^2\mathrm{Im}(\mathbf{h}_D^\dagger\mathbf{h}_D\mathbf{h}_U^\dagger)_{33}-\frac{i}{192\pi^2}\sum_{i=1,2}F_i^2g_i^2\Lambda^C_{Q,I}\mathcal{C}^1_{112}(x_\mu,x_i,x_{Q_I})-\frac{i}{48\pi^2}g_1^2\Lambda^C_{U,I}\mathcal{C}^1_{112}(x_\mu,x_1,x_{U_I}) \tag{A1}$$

with $F_1=1$, $F_2=3$, $T^U_Z=-T^D_Z=\frac{1}{2}$, $Y^B_U=\frac{4}{3}$, $Y^B_D=-\frac{2}{3}$ and $\hat{\mathbf{A}}_U=(1/\mu_{NP})(\mathbf{A}_U-\mu^*/\tan\beta\mathbf{h}_U)$, $\hat{\mathbf{A}}_D=(1/\mu_{NP})(\mathbf{A}_D-\mu^*\tan\beta\mathbf{h}_D)$, $\hat{\mathbf{A}}_E=(1/\mu_{NP})(\mathbf{A}_E-\mu^*\tan\beta\mathbf{h}_E)$, and $\hat{\mathbf{A}}'_F=\hat{\mathbf{A}}_F+(\mu^*/s_\beta c_\beta\mu_{NP})\mathbf{h}_F$ $(F=U,D,E)$. In the above expression, the sum with the generation indices I,J is implied.

APPENDIX B: THE WILSON COEFFICIENTS FOR OPERATORS $\mathcal{O}_{t\Phi i}$ ($I=1,2,3$)

The corresponding Wilson coefficients are

$$C_{t\Phi 1}=-\frac{1}{32\pi^2 x_H}c_\beta^2[\mathbf{h}_U(s_\beta^2\mathbf{h}_U^\dagger\mathbf{h}_U-c_\beta^2\mathbf{h}_D^\dagger\mathbf{h}_D)\mathbf{h}_U^\dagger]_{33}(1-\ln x_H)+\frac{\mathcal{X}^R_{U,IJK}}{72\pi^2}[g_1^2\mathcal{D}^1_{1121}(x_1,x_{U_I},x_{Q_J},x_{U_K})+3g_3^2\mathcal{D}^1_{1121}(x_3,x_{U_I},x_{Q_J},x_{U_K})]$$
$$+\frac{1}{64\pi^2}\sum_{q=U,D}\mathcal{X}^S_{q,IJK}\mathcal{D}^1_{1121}(x_\mu,x_{Q_I},x_{q_J},x_{Q_K})+\frac{1}{144\pi^2}g_1^4\mathcal{Z}_U^{\dagger 3I}\mathcal{Z}_U^{I3}[c_\beta^2Q_5(x_\mu,x_1,x_{U_I})+2\xi^S_{H1}Q_6(x_\mu,x_1,x_{U_I})$$
$$+s_\beta^2x_1Q_7(x_\mu,x_1,x_{U_I})]-\sum_{i=1,2}\frac{F_ig_i^2}{128\pi^2}(\mathbf{h}_U\mathcal{Z}_Q)_{3I}(\mathcal{Z}_Q^\dagger\mathbf{h}_U^\dagger)_{I3}[2\xi^S_{Hi}Q_6(x_i,x_\mu,x_{Q_I})+s_\beta^2Q_5(x_i,x_\mu,x_{Q_I})+c_\beta^2x_\mu Q_7(x_i,x_\mu,x_{Q_I})]$$
$$+\frac{1}{48\pi^2}g_1^2s_\beta[2\Gamma^U_{U,IJ}\mathcal{D}^0_{1111}(x_\mu,x_1,x_{Q_I},x_{U_J})-\Gamma^T_{U,IJ}U_5(x_\mu,x_{Q_I},x_1,x_{U_J})],$$

$$C_{t\Phi 2}=\frac{(\mathbf{h}_U\mathbf{h}_U^\dagger)_{33}}{256\pi^2x_H}c_\beta^2[4s_\beta^2(s_\beta^2-c_\beta^2)(2g_1^2+3g_2^2)\ln x_H-12s_\beta^4g_2^2+(c_\beta^4-7s_\beta^4)g_1^2]+\frac{g_1^2}{108\pi^2}(s_\beta^2-c_\beta^2)\mathcal{Z}_U^{\dagger 3I}\mathcal{Z}_U^{I3}[g_1^2\mathcal{B}^1_{3,1}(x_{U_I},x_1)$$
$$+3g_3^2\mathcal{B}^1_{3,1}(x_{U_I},x_3)]+\frac{c_\beta^2}{72\pi^2}\mathrm{Re}[\mathcal{Z}_U^{\dagger 3I}(\mathcal{Z}_U\mathbf{h}_U\mathbf{h}_U^\dagger\mathcal{Z}_U^\dagger)^{IJ}\mathcal{Z}_U^{J3}][g_1^2Q_9(x_{U_I},x_1,x_{U_J})+3g_3^2Q_9(x_{U_I},x_3,x_{U_J})]-\frac{1}{192\pi^2}g_1^2(s_\beta^2$$
$$-c_\beta^2)(\mathbf{h}_U\mathcal{Z}_Q)_{3I}(\mathcal{Z}_Q^\dagger\mathbf{h}_U^\dagger)_{I3}\mathcal{B}^1_{3,1}(x_{Q_I},x_\mu)+\frac{1}{16\pi^2}\mathrm{Re}[(\mathbf{h}_U\mathcal{Z}_Q)^{3I}(\mathcal{Z}_Q^\dagger(c_\beta^2\mathbf{h}_U^\dagger\mathbf{h}_U+s_\beta^2\mathbf{h}_D^\dagger\mathbf{h}_D)\mathcal{Z}_Q)^{IJ}(\mathcal{Z}_Q^\dagger\mathbf{h}_U^\dagger)^{J3}]Q_9(x_{Q_I},x_\mu,x_{Q_J})$$
$$-\frac{c_\beta^2}{64\pi^2x_H}[\mathbf{h}_U(s_\beta^2\mathbf{h}_U^\dagger\mathbf{h}_U+c_\beta^2\mathbf{h}_D^\dagger\mathbf{h}_D)\mathbf{h}_U^\dagger]_{33}+\frac{1}{64\pi^2}\sum_{q=U,D}\mathcal{X}^S_{q,IJK}U_1(x_\mu,x_{Q_I},x_{q_J},x_{Q_K})+\frac{1}{72\pi^2}\mathcal{X}^R_{U,IJK}[g_1^2U_1(x_i,x_{U_I},x_{Q_J},x_{U_K})$$
$$+3g_3^2U_1(x_3,x_{U_I},x_{Q_J},x_{U_K})]+\frac{1}{144\pi^2}g_1^4\mathcal{Z}_U^{\dagger 3I}\mathcal{Z}_U^{I3}[c_\beta^2\mathcal{C}^2_{122}(x_\mu,x_1,x_{U_I})+s_\beta^2\mathcal{C}^1_{122}(x_\mu,x_1,x_{U_I})+2\xi^S_{H1}\mathcal{C}^1_{122}(x_\mu,x_1,x_{U_I})]$$
$$+\sum_{i=1,2}\frac{F_ig_i^2}{128\pi^2}(h_U\mathcal{Z}_Q)_{3I}(\mathcal{Z}_Q^\dagger h_U^\dagger)_{I3}[c_\beta^2x_\mu\mathcal{C}^1_{212}(x_\mu,x_i,x_{Q_I})+s_\beta^2\mathcal{C}^2_{212}(x_\mu,x_i,x_{Q_I})+2\xi^S_{Hi}\mathcal{C}^1_{212}(x_\mu,x_i,x_{Q_I})]$$
$$-\frac{1}{48\pi^2}g_1^2s_\beta\Gamma^T_{U,IJ}U_5(x_\mu,x_1,x_{Q_I},x_{U_J}),$$

$$C_{t\Phi 3}=i\frac{c_\beta^2}{72\pi^2}\mathrm{Im}[\mathcal{Z}_U^{\dagger 3I}(\mathcal{Z}_U\mathbf{h}_U\mathbf{h}_U^\dagger\mathcal{Z}_U^\dagger)^{IJ}\mathcal{Z}_U^{J3}][g_1^2\mathcal{C}^1_{211}(x_{U_I},x_1,x_{U_J})+3g_3^2\mathcal{C}^1_{211}(x_{U_I},x_3,x_{U_J})]+\frac{i}{16\pi^2}\mathrm{Im}[(\mathbf{h}_U\mathcal{Z}_Q)^{3I}(\mathcal{Z}_Q^\dagger(c_\beta^2\mathbf{h}_U^\dagger\mathbf{h}_U$$
$$+s_\beta^2\mathbf{h}_D^\dagger\mathbf{h}_D)\mathcal{Z}_Q)^{IJ}(\mathcal{Z}_Q^\dagger\mathbf{h}_U^\dagger)^{J3}]\mathcal{C}^1_{211}(x_{Q_I},x_\mu,x_{Q_J})+i\frac{1}{36\pi^2}\mathcal{X}^A_{U,IJK}[g_1^2\mathcal{D}^1_{1211}(x_1,x_{U_I},x_{Q_J},x_{U_K})+3g_3^2\mathcal{D}^1_{1211}(x_3,x_{U_I},x_{Q_J},x_{U_K})]$$
$$+i\frac{1}{32\pi^2}\sum_{q=U,D}\mathcal{X}^B_{q,IJK}\mathcal{D}^1_{1211}(x_\mu,x_{Q_I},x_{q_J},x_{Q_K})+\frac{i}{36\pi^2}g_1^4\xi^B_{H1}\mathcal{Z}_U^{\dagger 3I}\mathcal{Z}_U^{I3}\mathcal{C}^0_{121}(x_\mu,x_1,x_{U_I})$$
$$+i\sum_{i=1,2}\frac{F_ig_i^2}{32\pi^2}\xi^B_{Hi}(h_U\mathcal{Z}_Q)_{3I}(\mathcal{Z}_Q^\dagger h_U^\dagger)_{I3}\mathcal{C}^0_{211}(x_\mu,x_i,x_{Q_I})+\frac{i}{48\pi^2}g_1^2s_\beta[2\Gamma^D_{U,IJ}\mathcal{D}^0_{1111}(x_\mu,x_1,x_{Q_I},x_{U_J})$$
$$-\Gamma^C_{U,IJ}U_5(x_\mu,x_{U_J},x_{Q_I},x_1)]. \tag{B1}$$

Just as in Appendix A, the sum with the generation indices I,J,K is implied.

APPENDIX C: THE WILSON COEFFICIENTS FOR OPERATORS $\mathcal{O}_{q\Phi i}$ ($I=1,\ldots,6$)

The Wilson coefficients for these operators are written as

$$
\begin{aligned}
C_{q\Phi 1} &= \frac{1}{32\pi^2 x_H} s_\beta^2 c_\beta^2 [(\mathbf{h}_U^\dagger \mathbf{h}_U \mathbf{h}_U^\dagger \mathbf{h}_U)_{33} - (\mathbf{h}_D^\dagger \mathbf{h}_D \mathbf{h}_D^\dagger \mathbf{h}_D)_{33}](1-\ln x_H) + \sum_{q=U,D} (-1)^{1/2-T_Z^q} \mathcal{X}^T_{q,IJK} \Bigg[\sum_{i=1,2} \frac{F_i^2 g_i^2}{2304\pi^2} \mathcal{D}^1_{1121}(x_i, x_{Q_I}, x_{q_J}, x_{Q_K}) \\
&+ \frac{g_3^2}{48\pi^2} \mathcal{D}^1_{1121}(x_3, x_{Q_I}, x_{q_J}, x_{Q_K}) \Bigg] - \frac{1}{64\pi^2} \sum_{q=U,D} (-1)^{1/2-T_Z^q} \mathcal{X}^U_{q,IJK} \mathcal{D}^1_{1121}(x_\mu, x_{q_I}, x_{Q_J}, x_{q_K}) + \frac{g_1^4}{2304\pi^2} (c_\beta^2 \\
&- s_\beta^2) \mathcal{Z}_Q^{3I} \mathcal{Z}_Q^{\dagger I3} Q_8(x_\mu, x_1, x_{Q_I}) + \sum_{i=1,2} \frac{F_i g_i^2}{256\pi^2} (\mathbf{h}_U^\dagger \mathcal{Z}_U^\dagger)_{3I} (\mathcal{Z}_U \mathbf{h}_U)_{I3} [s_\beta^2 Q_{10}(x_\mu, x_i, x_{U_I}) - 4\xi^S_{Hi} \mathcal{C}^0_{211}(x_\mu, x_i, x_{U_I}) \\
&+ \eta^R_{Hi} Q_7(x_i, x_\mu, x_{U_I})] - \sum_{i=1,2} \frac{F_i g_i^2}{256\pi^2} (\mathbf{h}_D^\dagger \mathcal{Z}_D^\dagger)_{3I} (\mathcal{Z}_D \mathbf{h}_D)_{I3} [c_\beta^2 Q_{10}(x_\mu, x_i, x_{D_I}) - 4\xi^S_{Hi} \mathcal{C}^0_{211}(x_\mu, x_i, x_{D_I}) + \eta^S_{Hi} Q_7(x_i, x_\mu, x_{D_I})] \\
&+ \frac{g_1^2}{256\pi^2} \sum_{q=U,D} Y_q^B [2\Gamma^W_{q,IJ} \mathcal{D}^0_{1111}(x_\mu, x_1, x_{q_I}, x_{Q_J}) - \Gamma^{V,1}_{q,IJ} U_5(x_\mu, x_{q_I}, x_1, x_{Q_J})] - \frac{3g_2^2}{128\pi^2} \sum_{q=U,D} [2\Gamma^W_{q,IJ} \mathcal{D}^0_{1111}(x_\mu, x_2, x_{q_I}, x_{Q_J}) \\
&- \Gamma^{V,2}_{q,IJ} U_5(x_\mu, x^2_{q_I}, x_2, x_{Q_J})],
\end{aligned}
$$

$$
\begin{aligned}
C_{q\Phi 2} &= \sum_{q=U,D} \mathcal{X}^T_{q,IJK} \Bigg[\sum_{i=1,2} (-1)^i \frac{F_i^2 g_i^2}{2304\pi^2} \mathcal{D}^1_{1121}(x_i, x_{Q_I}, x_{q_J}, x_{Q_K}) - \frac{g_3^2}{48\pi^2} \mathcal{D}^1_{1121}(x_3, x_{Q_I}, x_{q_J}, x_{Q_K}) \Bigg] + \frac{g_2^4}{128\pi^2} (s_\beta^2 \\
&- c_\beta^2) \mathcal{Z}_Q^{3I} \mathcal{Z}_Q^{\dagger I3} Q_8(x_\mu, x_2, x_{Q_I}) - \frac{g_1^2 g_2^2}{384\pi^2} (s_\beta^2 - c_\beta^2) \mathcal{Z}_Q^{3I} \mathcal{Z}_Q^{\dagger I3} [U_2(x_\mu, x_1, x_2, x_{Q_I}) + \sqrt{x_1 x_2} \cos(\varphi_2 - \varphi_1) U_3(x_\mu, x_1, x_2, x_{Q_I})] \\
&+ \sum_{i=1,2} (-1)^i \frac{g_i^2}{256\pi^2} (\mathbf{h}_U^\dagger \mathcal{Z}_U^\dagger)_{3I} (\mathcal{Z}_U \mathbf{h}_U)_{I3} [s_\beta^2 Q_{10}(x_\mu, x_i, x_{U_I}) - 4\xi^S_{Hi} \mathcal{C}^0_{211}(x_\mu, x_i, x_{U_I}) + \eta^R_{Hi} Q_7(x_i, x_\mu, x_{U_I})] \\
&+ \sum_{i=1,2} (-1)^i \frac{g_i^2}{256\pi^2} (\mathbf{h}_D^\dagger \mathcal{Z}_D^\dagger)_{3I} (\mathcal{Z}_D \mathbf{h}_D)_{I3} [c_\beta^2 Q_{10}(x_\mu, x_i, x_{D_I}) - 4\xi^S_{Hi} \mathcal{C}^0_{211}(x_\mu, x_i, x_{D_I}) + \eta^S_{Hi} Q_7(x_i, x_\mu, x_{D_I})] \\
&- \frac{g_1^2}{256\pi^2} \sum_{q=U,D} (-1)^{1/2-T_Z^q} Y_q^B [2\Gamma^W_{q,IJ} \mathcal{D}^0_{1111}(x_\mu, x_1, x_{q_I}, x_{Q_J}) - \Gamma^{V,1}_{q,IJ} U_5(x_\mu, x_{q_I}, x_1, x_{Q_J})] \\
&- \frac{g_2^2}{128\pi^2} \sum_{q=U,D} (-1)^{1/2-T_Z^q} [2\Gamma^W_{q,IJ} \mathcal{D}^0_{1111}(x_\mu, x_2, x_{q_I}, x_{Q_J}) - \Gamma^{V,2}_{q,IJ} U_5(x_\mu, x_{q_I}, x_2, x_{Q_J})],
\end{aligned}
$$

$$
\begin{aligned}
C_{q\Phi 3} &= 3\frac{g_1^2 + g_2^2}{128\pi^2} s_\beta^2 c_\beta^2 (s_\beta^2 - c_\beta^2) [(\mathbf{h}_U^\dagger \mathbf{h}_U)_{33} - (\mathbf{h}_D^\dagger \mathbf{h}_D)_{33}] \frac{1+\ln x_H}{x_H} + \frac{1}{256\pi^2 x_H} (g_1^2 - 6 s_\beta^2 c_\beta^2 (g_1^2 + g_2^2)) [c_\beta^2 (\mathbf{h}_U^\dagger \mathbf{h}_U)_{33} + s_\beta^2 (\mathbf{h}_D^\dagger \mathbf{h}_D)_{33}] \\
&+ \frac{g_1^2 (s_\beta^2 - c_\beta^2)}{6912\pi^2} \mathcal{Z}_Q^{3I} \mathcal{Z}_Q^{\dagger I3} \Bigg[\sum_{i=1,2} F_i^3 g_i^2 \mathcal{B}^1_{3,1}(x_{Q_I}, x_1) + 48 g_3^2 \mathcal{B}^1_{3,1}(x_{Q_I}, x_3) \Bigg] + \frac{1}{2304\pi^2} \Lambda^S_{IJ} \Bigg[\sum_{i=1,2} F_i^2 g_i^2 Q_9(x_{Q_I}, x_i, x_{Q_J}) \\
&+ 48 g_3^2 Q_9(x_{Q_I}, x_3, x_{Q_J}) \Bigg] + \frac{1}{128\pi^2} \sum_{q=U,D} [Y_q^B g_1^2 (s_\beta^2 - c_\beta^2) (\mathbf{h}_q^\dagger \mathcal{Z}_q^\dagger)_{3I} (\mathcal{Z}_q \mathbf{h}_q)_{I3} \mathcal{B}^1_{3,1}(x_{q_I}, x_\mu) + 2\Lambda^U_{q,IJ} Q_9(x_{q_I}, x_\mu, x_{q_J})] \\
&- \frac{1}{64\pi^2 x_H} s_\beta^2 c_\beta^2 [(\mathbf{h}_U^\dagger \mathbf{h}_U \mathbf{h}_U^\dagger \mathbf{h}_U)_{33} + (\mathbf{h}_U^\dagger \mathbf{h}_U \mathbf{h}_U^\dagger \mathbf{h}_U)_{33}] + \sum_{q=U,D} \mathcal{X}^T_{q,IJK} \Bigg[\sum_{i=1,2} \frac{F_i^3 g_i^2}{2304\pi^2} U_1(x_i, x_{Q_I}, x_{q_J}, x_{Q_K}) \\
&+ \frac{g_3^2}{48\pi^2} U_1(x_3, x_{Q_I}, x_{U_J}, x_{Q_K}) \Bigg] + \frac{g_1^4}{2304\pi^2} \mathcal{Z}_Q^{3I} \mathcal{Z}_Q^{\dagger I3} [\mathcal{C}^2_{122}(x_\mu, x_1, x_{Q_I}) + \eta^T_{H1} \mathcal{C}^1_{122}(x_\mu, x_1, x_{Q_I})]
\end{aligned}
$$

$$+\sum_{i=1,2}\frac{F_i g_i^2}{256\pi^2}(\mathbf{h}_U^\dagger \mathcal{Z}_U^\dagger)_{3I}(\mathcal{Z}_U \mathbf{h}_U)_{I3}[\,\eta^R_{Hi}\mathcal{C}^1_{212}(x_\mu,x_i,x_{U_I})+s_\beta^2\mathcal{C}^2_{212}(x_\mu,x_i,x_{U_I})]$$

$$+\sum_{i=1,2}\frac{F_i g_i^2}{256\pi^2}(\mathbf{h}_D^\dagger \mathcal{Z}_D^\dagger)_{3I}(\mathcal{Z}_D \mathbf{h}_D)_{I3}[\,\eta^S_{Hi}\mathcal{C}^1_{212}(x_\mu,x_i,x_{D_I})+c_\beta^2\mathcal{C}^2_{212}(x_\mu,x_i,x_{D_I})]+\frac{1}{64\pi^2}\sum_{q=U,D}\mathcal{X}^U_{q,IJK}U_1(x_\mu,x_{q_I},x_{Q_J},x_{q_K})$$

$$+\frac{g_1^2}{256\pi^2}\sum_{q=U,D}(-1)^{1/2-T_Z^q}Y_q^B\Gamma^{V,1}_{q,IJ}U_5(x_\mu,x_1,x_{q_I},x_{Q_J})-\frac{3g_2^2}{128\pi^2}\sum_{q=U,D}(-1)^{1/2-T_Z^q}\Gamma^{V,2}_{q,IJ}U_5(x_\mu,x_2,x_{q_I},x_{Q_J}),$$

$$C_{q\Phi 4}=-\frac{g_1^2+g_2^2}{128\pi^2}s_\beta^2c_\beta^2(s_\beta^2-c_\beta^2)[(\mathbf{h}_U^\dagger\mathbf{h}_U)_{33}+(\mathbf{h}_D^\dagger\mathbf{h}_D)_{33}]\frac{1+\ln x_H}{x_H}-\frac{1}{256\pi^2 x_H}(g_2^2-2s_\beta^2c_\beta^2(g_1^2+g_2^2))[c_\beta^2(\mathbf{h}_U^\dagger\mathbf{h}_U)_{33}-s_\beta^2(\mathbf{h}_D^\dagger\mathbf{h}_D)_{33}]$$

$$-\frac{g_2^2}{2304\pi^2}(s_\beta^2-c_\beta^2)\mathcal{Z}_Q^{3I}\mathcal{Z}_Q^{\dagger I3}\left[\sum_{i=1,2}(-1)^i F_i^2 g_i^2\mathcal{B}^1_{3,1}(x_{Q_I},x_i)-48g_3^2\mathcal{B}^1_{3,1}(x_{Q_I},x_3)\right]+\frac{1}{2304\pi^2}$$

$$\times\Lambda^S_{IJ}\left[\sum_{i=1,2}(-1)^{i-1}F_i^2g_i^2Q_9(x_{Q_I},x_i,x_{Q_J})+48g_3^2Q_9(x_{Q_I},x_3,x_{Q_J})\right]+\frac{1}{2304\pi^2}\sum_{q=U,D}(-1)^{1/2-T_Z^q}\mathcal{X}^T_{q,IJK}$$

$$\times\left[\sum_{i=1,2}(-1)^iF_i^2g_i^2U_1(x_i,x_{Q_I},x_{q_J},x_{Q_K})-48g_3^2U_1(x_3,x_{Q_I},x_{q_J},x_{Q_K})\right]-\frac{g_2^2}{384\pi^2}\mathcal{Z}_Q^{3I}\mathcal{Z}_Q^{\dagger I3}\{3g_2^2[\mathcal{C}^2_{122}(x_\mu,x_2,x_{Q_I})$$

$$+\eta^T_{H2}\mathcal{C}^1_{122}(x_\mu,x_2,x_{Q_I})]-g_1^2[\mathcal{D}^2_{1112}(x_\mu,x_1,x_2,x_{Q_I})+(\sqrt{x_1x_2}\cos(\varphi_2-\varphi_1)+2[\xi^S_{H1}+\xi^S_{H2}])\mathcal{D}^1_{1112}(x_\mu,x_1,x_2,x_{Q_I})]\}$$

$$+\sum_{i=1,2}(-1)^i\frac{g_i^2}{256\pi^2}(\mathbf{h}_U^\dagger\mathcal{Z}_U^\dagger)_{3I}(\mathcal{Z}_U\mathbf{h}_U)_{I3}[\,\eta^R_{Hi}\mathcal{C}^1_{212}(x_\mu,x_i,x_{U_I})+s_\beta^2\mathcal{C}^2_{212}(x_\mu,x_i,x_{U_I})]$$

$$-\sum_{i=1,2}(-1)^i\frac{g_1^2}{256\pi^2}(\mathbf{h}_D^\dagger\mathcal{Z}_D^\dagger)_{3I}(\mathcal{Z}_D\mathbf{h}_D)_{I3}[\,\eta^S_{Hi}\mathcal{C}^1_{212}(x_\mu,x_i,x_{D_I})+c_\beta^2\mathcal{C}^2_{212}(x_\mu,x_i,x_{D_I})]$$

$$-\frac{g_1^2}{256\pi^2}\sum_{q=U,D}Y_q^B\Gamma^{V,1}_{q,IJ}U_5(x_\mu,x_1,x_{q_I},x_{Q_J})-\frac{g_2^2}{128\pi^2}\sum_{q=U,D}\Gamma^{V,2}_{q,IJ}U_5(x_\mu,x_2,x_{q_I},x_{Q_J}),$$

$$C_{q\Phi 5}=\frac{1}{2304\pi^2}\left[\Lambda^B_{IJ}\left(\sum_{i=1,2}g_i^2F_i^3\mathcal{C}^1_{112}(x_{Q_I},x_i,x_{Q_J})+48g_3^2\mathcal{C}^1_{112}(x_{Q_I},x_3,x_{Q_J})\right)+36\sum_{q=U,D}\Lambda^D_{q,IJ}\mathcal{C}^1_{112}(x_{q_I},x_\mu,x_{q_J})\right]$$

$$+\frac{i}{1152\pi^2}\sum_{q=U,D}\sum_{i=1,2}\mathcal{X}^{\mathcal{C}}_{q,IJK}[F_i^3g_i^2\mathcal{D}^1_{1211}(x_i,x_{Q_I},x_{q_J},x_{Q_K})+48g_3^2\mathcal{D}^1_{1211}(x_3,x_{Q_I},x_{q_J},x_{Q_K})]$$

$$+\frac{i}{32\pi^2}\sum_{q=U,D}\mathcal{X}^{\mathcal{D}}_{q,IJK}\mathcal{D}^1_{1211}(x_\mu,x_{q_I},x_{Q_J},x_{q_K})-i\frac{g_1^4}{288\pi^2}\xi^B_{H1}\mathcal{Z}_Q^{3I}\mathcal{Z}_Q^{\dagger I3}\mathcal{C}^0_{121}(x_\mu,x_1,x_{Q_I})$$

$$-\frac{i}{64\pi^2}\sum_{q=U,D}\sum_{i=1,2}F_ig_i^2\xi^B_{Hi}(\mathbf{h}_q^\dagger\mathcal{Z}_q^\dagger)_{3I}(\mathcal{Z}_q\mathbf{h}_q)_{I3}\mathcal{C}^0_{211}(x_\mu,x_i,x_{q_I})+i\frac{g_1^2}{256\pi^2}\sum_{q=U,D}(-1)^{1/2-T_Z^q}Y_q^B[\Gamma^{E,1}_{q,IJ}U_5(x_\mu,x_{Q_J},x_{q_I},x_1)$$

$$-2\Gamma^F_{q,IJ}\mathcal{D}^0_{1111}(x_\mu,x_1,x_{q_I},x_{Q_J})]-i\frac{3g_2^2}{128\pi^2}\sum_{q=U,D}(-1)^{1/2-T_Z^q}[\Gamma^{E,2}_{q,IJ}U_5(x_\mu,x_{Q_J},x_{q_I},x_2)-2\Gamma^F_{q,IJ}\mathcal{D}^0_{1111}(x_\mu,x_2,x_{q_I},x_{Q_J})],$$

$$C_{q\Phi 6}=\frac{\Lambda^B_{IJ}}{2304\pi^2}\left(\sum_{i=1,2}(-1)^{i-1}g_i^2F_i^3\mathcal{C}^1_{112}(x_{Q_I},x_i,x_{Q_J})+48g_3^2\mathcal{C}^1_{112}(x_{Q_I},x_3,x_{Q_J})\right)+\frac{i}{1152\pi^2}\sum_{q=U,D}(-1)^{1/2-T_Z^q}$$

$$\times\left[\sum_{i=1,2}(-1)^iF_i^2g_i^2\mathcal{X}^{\mathcal{C}}_{q,IJK}\mathcal{D}^1_{1211}(x_1,x_{Q_I},x_{q_J},x_{Q_K})-48g_3^2\mathcal{X}^{\mathcal{C}}_{q,IJK}\mathcal{D}^1_{1211}(x_3,x_{Q_I},x_{q_J},x_{Q_K})\right]+i\frac{g_2^4}{16\pi^2}\xi^B_{H2}\mathcal{Z}_Q^{3I}\mathcal{Z}_Q^{\dagger I3}\mathcal{C}^0_{121}(x_\mu,x_2,x_{Q_I})$$

$$-i\frac{g_1^2g_2^2}{384\pi^2}\mathcal{Z}_Q^{3I}\mathcal{Z}_Q^{\dagger I3}[4(\xi^B_{H1}+\xi^B_{H2})\mathcal{D}^0_{1111}(x_\mu,x_1,x_2,x_{Q_I})-((c_\beta^2-s_\beta^2)\sqrt{x_1x_2}\sin(\varphi_2-\varphi_1)-2[\xi^B_{H1}-\xi^B_{H2}])U_4(x_\mu,x_1,x_2,x_{Q_I})]$$

$$-\frac{1}{64\pi^2}\sum_{q=U,D}(-1)^{1/2-T_Z^q}\sum_{i=1,2}(-1)^i g_i^2\xi_{Hi}^B(\mathbf{h}_U^\dagger\mathcal{Z}_U^\dagger)_{3I}(\mathcal{Z}_U\mathbf{h}_U)_{I3}\mathcal{C}_{211}^0(x_\mu,x_i,x_{U_I})$$

$$-i\frac{g_1^2}{256\pi^2}\sum_{q=U,D}Y_q^B[\Gamma_{q,IJ}^{E,1}U_5(x_\mu,x_{Q_J},x_{q_I},x_1)-2\Gamma_{q,IJ}^F\mathcal{D}_{1111}^0(x_\mu,x_1,x_{q_I},x_{Q_J})]-i\frac{g_2^2}{128\pi^2}\sum_{q=U,D}[\Gamma_{q,IJ}^{E,2}U_5(x_\mu,x_{Q_J},x_{q_I},x_2)$$

$$-2\Gamma_{q,IJ}^F\mathcal{D}_{1111}^0(x_\mu,x_2,x_{q_I},x_{Q_J})]. \tag{C1}$$

APPENDIX D: THE COUPLING CONSTANTS AND LOOP FUNCTIONS

The loop functions are defined as

$$P_H(x,y)=-\mathcal{B}_{1,3}^0(x,y)+3\mathcal{B}_{1,4}^1(x,y)-\frac{2}{3}\mathcal{B}_{1,5}^2(x,y),$$

$$Q_1(x,y,z)=\mathcal{C}_{122}^1(x,y,z)+y\mathcal{C}_{131}^0(x,y,z)+z\mathcal{C}_{113}^0(x,y,z),$$

$$Q_2(x,y,z)=\mathcal{C}_{122}^2(x,y,z)+y\mathcal{C}_{131}^1(x,y,z)+z\mathcal{C}_{113}^1(x,y,z),$$

$$Q_3(x,y,z)=\mathcal{C}_{122}^1(x,y,z)+x\mathcal{C}_{131}^0(x,y,z)+z\mathcal{C}_{113}^0(x,y,z),$$

$$Q_4(x,y,z)=\mathcal{C}_{122}^2(x,y,z)+x\mathcal{C}_{131}^1(x,y,z)+z\mathcal{C}_{113}^1(x,y,z),$$

$$Q_5(x,y,z)=\mathcal{C}_{122}^2(x,y,z)+2\mathcal{C}_{131}^2(x,y,z)-4\mathcal{C}_{121}^1(x,y,z),$$

$$Q_6(x,y,z)=\mathcal{C}_{122}^1(x,y,z)+2\mathcal{C}_{131}^1(x,y,z)+2\mathcal{C}_{121}^0(x,y,z),$$

$$Q_7(x,y,z)=\mathcal{C}_{122}^1(x,y,z)+2\mathcal{C}_{131}^1(x,y,z),$$

$$Q_8(x,y,z)=4\mathcal{C}_{121}^1(x,y,z)-2\mathcal{C}_{131}^2(x,y,z)-\mathcal{C}_{122}^2(x,y,z)+y(2\mathcal{C}_{131}^1(x,y,z)+\mathcal{C}_{122}^1(x,y,z)),$$

$$Q_9(x,y,z)=\mathcal{C}_{211}^1(x,y,z)+\mathcal{C}_{112}^1(x,y,z),$$

$$Q_{10}(x,y,z)=\mathcal{C}_{212}^2(x,y,z)+2\mathcal{C}_{311}^2(x,y,z)-4\mathcal{C}_{211}^1(x,y,z),$$

$$U_1(x,y,z,w)=2\mathcal{D}_{1211}^1(x,y,z,w)+\mathcal{D}_{1121}^1(x,y,z,w),$$

$$U_2(x,y,z,w)=4\mathcal{D}_{1111}^1(x,y,z,w)-\mathcal{D}_{1211}^2(x,y,z,w)-\mathcal{D}_{1121}^2(x,y,z,w)-\mathcal{D}_{1112}^2(x,y,z,w),$$

$$U_3(x,y,z,w)=\mathcal{D}_{1211}^1(x,y,z,w)+\mathcal{D}_{1121}^1(x,y,z,w)+\mathcal{D}_{1112}^1(x,y,z,w),$$

$$U_4(x,y,z,w)=\mathcal{D}_{1211}^1(x,y,z,w)-\mathcal{D}_{1121}^1(x,y,z,w),$$

$$U_5(x,y,z,w)=\mathcal{D}_{1121}^1(x,y,z,w)+\mathcal{D}_{1112}^1(x,y,z,w). \tag{D1}$$

The coupling constants are

$$\xi_{Hi}^R=\sqrt{x_\mu x_i}((c_\beta^2-s_\beta^2)\mathrm{Re}(\mathbf{h}_U)_{33}\cos(\varphi_\mu+\varphi_i)-\mathrm{Im}(\mathbf{h}_U)_{33}\sin(\varphi_\mu+\varphi_i)) \quad (i=1,2),$$

$$\xi_{Hi}^A=\sqrt{x_\mu x_i}(\mathrm{Im}(\mathbf{h}_U)_{33}(c_\beta^2-s_\beta^2)\cos(\varphi_\mu+\varphi_i)+\mathrm{Re}(\mathbf{h}_U)_{33}\sin(\varphi_\mu+\varphi_i)) \quad (i=1,2),$$

$$\xi_{Hi}^S=s_\beta c_\beta\sqrt{x_\mu x_i}\cos(\varphi_\mu+\varphi_i) \quad (i=1,2),$$

$$\xi_{Hi}^B=s_\beta c_\beta\sqrt{x_\mu x_i}\sin(\varphi_\mu+\varphi_i) \quad (i=1,2),$$

$$\eta^R_{Hi} = c_\beta^2 x_\mu + 2 s_\beta c_\beta \sqrt{x_\mu x_i}\cos(\varphi_\mu + \varphi_i) \quad (i=1,2),$$

$$\eta^S_{Hi} = s_\beta^2 x_\mu + 2 s_\beta c_\beta \sqrt{x_\mu x_i}\cos(\varphi_\mu + \varphi_i) \quad (i=1,2),$$

$$\eta^T_{Hi} = x_i + 4 s_\beta c_\beta \sqrt{x_\mu x_i}\cos(\varphi_\mu + \varphi_i) \quad (i=1,2),$$

$$\Lambda^{R,i}_{U,I} = \sqrt{x_\mu x_i}\{\mathrm{Re}[\mathcal{Z}_Q^{3I}(\mathcal{Z}_Q^\dagger \mathbf{h}_U^\dagger)^{I3}]\cos(\varphi_\mu+\varphi_i) - \mathrm{Im}[\mathcal{Z}_Q^{3I}(\mathcal{Z}_Q^\dagger \mathbf{h}_U^\dagger)^{I3}]\sin(\varphi_\mu+\varphi_i)\} \quad (i=1,\ \text{or}\ 2),$$

$$\Lambda^{A,i}_{U,I} = \sqrt{x_\mu x_i}\{\mathrm{Re}[\mathcal{Z}_Q^{3I}(\mathcal{Z}_Q^\dagger \mathbf{h}_U^\dagger)^{I3}]\sin(\varphi_\mu+\varphi_i) + \mathrm{Im}[\mathcal{Z}_Q^{3I}(\mathcal{Z}_Q^\dagger \mathbf{h}_U^\dagger)^{I3}]\cos(\varphi_\mu+\varphi_i)\} \quad (i=1,\ \text{or}\ 2),$$

$$\Lambda^S_{IJ} = \mathrm{Re}\{\mathcal{Z}_Q^{3I}[\mathcal{Z}_Q^\dagger c_\beta^2 (\mathbf{h}_U^\dagger \mathbf{h}_U)_{33} + s_\beta^2 (\mathbf{h}_D^\dagger \mathbf{h}_D)\mathcal{Z}_Q]^{IJ}\mathcal{Z}_Q^{\dagger J3}\},$$

$$\Lambda^B_{IJ} = \mathrm{Im}\{\mathcal{Z}_Q^{3I}[\mathcal{Z}_Q^\dagger c_\beta^2 (\mathbf{h}_U^\dagger \mathbf{h}_U)_{33} + s_\beta^2 (\mathbf{h}_D^\dagger \mathbf{h}_D)\mathcal{Z}_Q]^{IJ}\mathcal{Z}_Q^{\dagger J3}\},$$

$$\Lambda^T_{U,I} = \mathrm{Re}((\mathbf{h}_U^\dagger \mathcal{Z}_U^\dagger)^{3I}\mathcal{Z}_U^{I3}), \quad \Lambda^C_{U,I} = \mathrm{Im}((\mathbf{h}_U^\dagger \mathcal{Z}_U^\dagger)^{3I}\mathcal{Z}_U^{I3}),$$

$$\Lambda^T_{Q,I} = \mathrm{Re}(\mathcal{Z}_Q^{3I}(\mathcal{Z}_Q^\dagger \mathbf{h}_U^\dagger)^{I3}), \quad \Lambda^C_{Q,I} = \mathrm{Im}(\mathcal{Z}_Q^{3I}(\mathcal{Z}_Q^\dagger \mathbf{h}_U^\dagger)^{I3}),$$

$$\Lambda^U_{U,IJ} = c_\beta^2 \mathrm{Re}[(\mathcal{Z}_U \mathbf{h}_U)^{\dagger 3I}(\mathcal{Z}_U \mathbf{h}_U \mathbf{h}_U^\dagger \mathcal{Z}_U^\dagger)^{IJ}(\mathcal{Z}_U \mathbf{h}_U)^{J3}],$$

$$\Lambda^D_{U,IJ} = c_\beta^2 \mathrm{Im}[(\mathcal{Z}_U \mathbf{h}_U)^{\dagger 3I}(\mathcal{Z}_U \mathbf{h}_U \mathbf{h}_U^\dagger \mathcal{Z}_U^\dagger)^{IJ}(\mathcal{Z}_U \mathbf{h}_U)^{J3}],$$

$$\Lambda^U_{D,IJ} = s_\beta^2 \mathrm{Re}[(\mathcal{Z}_D \mathbf{h}_D)^{\dagger 3I}(\mathcal{Z}_D \mathbf{h}_D \mathbf{h}_D^\dagger \mathcal{Z}_D^\dagger)^{IJ}(\mathcal{Z}_D \mathbf{h}_D)^{J3}],$$

$$\Lambda^D_{D,IJ} = s_\beta^2 \mathrm{Im}[(\mathcal{Z}_D \mathbf{h}_D)^{\dagger 3I}(\mathcal{Z}_D \mathbf{h}_D \mathbf{h}_D^\dagger \mathcal{Z}_D^\dagger)^{IJ}(\mathcal{Z}_D \mathbf{h}_D)^{J3}],$$

$$\Gamma^{R,i}_{U,IJ} = \sqrt{x_i}\{\mathrm{Re}[\mathcal{Z}_Q^{3I}(\mathcal{Z}_U \hat{\mathbf{A}}_U \mathcal{Z}_Q)^{\dagger IJ}\mathcal{Z}_U^{J3}]\cos\varphi_i - \mathrm{Im}[\mathcal{Z}_Q^{3I}(\mathcal{Z}_U \hat{\mathbf{A}}_U \mathcal{Z}_Q)^{\dagger IJ}\mathcal{Z}_U^{J3}]\sin\varphi_i\} \quad (i=1,\ \text{or}\ 3),$$

$$\Gamma^{A,i}_{U,IJ} = \sqrt{x_i}\{\mathrm{Re}[\mathcal{Z}_Q^{3I}(\mathcal{Z}_U \hat{\mathbf{A}}_U \mathcal{Z}_Q)^{\dagger IJ}\mathcal{Z}_U^{J3}]\sin\varphi_i + \mathrm{Im}[\mathcal{Z}_Q^{3I}(\mathcal{Z}_U \hat{\mathbf{A}}_U \mathcal{Z}_Q)^{\dagger IJ}\mathcal{Z}_U^{J3}]\cos\varphi_i\} \quad (i=1,\ \text{or}\ 3),$$

$$\Gamma^S_{U,IJ} = \sqrt{x_\mu}\{\mathrm{Re}[(\mathbf{h}_D^\dagger \mathcal{Z}_D^\dagger)^{3I}(\mathcal{Z}_D \hat{\mathbf{A}}_D \mathcal{Z}_Q)^{IJ}(\mathcal{Z}_Q^\dagger \mathbf{h}_U^\dagger)^{J3}]\cos\varphi_\mu - \mathrm{Im}[(\mathbf{h}_D^\dagger \mathcal{Z}_D^\dagger)^{3I}(\mathcal{Z}_D \hat{\mathbf{A}}_D \mathcal{Z}_Q)^{IJ}(\mathcal{Z}_Q^\dagger \mathbf{h}_U^\dagger)^{J3}]\sin\varphi_\mu\},$$

$$\Gamma^B_{U,IJ} = \sqrt{x_\mu}\{\mathrm{Re}[(\mathbf{h}_D^\dagger \mathcal{Z}_D^\dagger)^{3I}(\mathcal{Z}_D \hat{\mathbf{A}}_D \mathcal{Z}_Q)^{IJ}(\mathcal{Z}_Q^\dagger \mathbf{h}_U^\dagger)^{J3}]\sin\varphi_\mu + \mathrm{Im}[(\mathbf{h}_D^\dagger \mathcal{Z}_D^\dagger)^{3I}(\mathcal{Z}_D \hat{\mathbf{A}}_D \mathcal{Z}_Q)^{IJ}(\mathcal{Z}_Q^\dagger \mathbf{h}_U^\dagger)^{J3}]\cos\varphi_\mu\},$$

$$\Gamma^T_{U,IJ} = \mathrm{Re}[(\mathbf{h}_U \mathcal{Z}_Q)_{3I}(\mathcal{Z}_U \hat{\mathbf{A}}_U \mathcal{Z}_Q)^\dagger_{IJ}\mathcal{Z}_U^{J3}][c_\beta\sqrt{x_\mu}\cos\varphi_\mu + s_\beta\sqrt{x_1}\cos\varphi_1] + \mathrm{Im}[(\mathbf{h}_U \mathcal{Z}_Q)_{3I}(\mathcal{Z}_U \hat{\mathbf{A}}_U \mathcal{Z}_Q)^\dagger_{IJ}\mathcal{Z}_U^{J3}]$$
$$\times[c_\beta\sqrt{x_\mu}\sin\varphi_\mu - s_\beta\sqrt{x_1}\sin\varphi_1],$$

$$\Gamma^U_{U,IJ} = c_\beta\sqrt{x_\mu}\{\mathrm{Re}[(\mathbf{h}_U \mathcal{Z}_Q)_{3I}(\mathcal{Z}_U \hat{\mathbf{A}}_U \mathcal{Z}_Q)^\dagger_{IJ}\mathcal{Z}_U^{J3}]\cos\varphi_\mu + \mathrm{Im}[(\mathbf{h}_U \mathcal{Z}_Q)_{3I}(\mathcal{Z}_U \hat{\mathbf{A}}_U \mathcal{Z}_Q)^\dagger_{IJ}\mathcal{Z}_U^{J3}]\sin\varphi_\mu\},$$

$$\Gamma^C_{U,IJ} = \mathrm{Im}[(\mathbf{h}_U \mathcal{Z}_Q)_{3I}(\mathcal{Z}_U \hat{\mathbf{A}}_U \mathcal{Z}_Q)^\dagger_{IJ}\mathcal{Z}_U^{J3}][c_\beta\sqrt{x_\mu}\cos\varphi_\mu + s_\beta\sqrt{x_1}\cos\varphi_1] - \mathrm{Re}[(\mathbf{h}_U \mathcal{Z}_Q)_{3I}(\mathcal{Z}_U \hat{\mathbf{A}}_U \mathcal{Z}_Q)^\dagger_{IJ}\mathcal{Z}_U^{J3}]$$
$$\times[c_\beta\sqrt{x_\mu}\sin\varphi_\mu - s_\beta\sqrt{x_1}\sin\varphi_1],$$

$$\Gamma^D_{U,IJ} = c_\beta\sqrt{x_\mu}\{\mathrm{Im}[(\mathbf{h}_U \mathcal{Z}_Q)_{3I}(\mathcal{Z}_U \hat{\mathbf{A}}_U \mathcal{Z}_Q)^\dagger_{IJ}\mathcal{Z}_U^{J3}]\cos\varphi_\mu - \mathrm{Re}[(\mathbf{h}_U \mathcal{Z}_Q)_{3I}(\mathcal{Z}_U \hat{\mathbf{A}}_U \mathcal{Z}_Q)^\dagger_{IJ}\mathcal{Z}_U^{J3}]\sin\varphi_\mu\},$$

$$\Gamma^{V,i}_{U,IJ} = \mathrm{Re}[(\mathbf{h}_U^\dagger \mathcal{Z}_U^\dagger)_{3I}(\mathcal{Z}_U \hat{\mathbf{A}}_U \mathcal{Z}_Q)_{IJ}\mathcal{Z}_Q^{\dagger J3}]s_\beta(c_\beta\sqrt{x_\mu}\cos\varphi_\mu + s_\beta\sqrt{x_i}\cos\varphi_i)$$
$$- \mathrm{Im}[(\mathbf{h}_U^\dagger \mathcal{Z}_U^\dagger)_{3I}(\mathcal{Z}_U \hat{\mathbf{A}}_U \mathcal{Z}_Q)_{IJ}\mathcal{Z}_Q^{\dagger J3}]s_\beta(c_\beta\sqrt{x_\mu}\sin\varphi_\mu - s_\beta\sqrt{x_i}\sin\varphi_i) \quad (i=1,2),$$

$$\Gamma^{E,i}_{U,IJ} = \mathrm{Re}[(\mathbf{h}_U^\dagger \mathcal{Z}_U^\dagger)_{3I}(\mathcal{Z}_U \hat{\mathbf{A}}_U \mathcal{Z}_Q)_{IJ}\mathcal{Z}_Q^{\dagger J3}]s_\beta(c_\beta\sqrt{x_\mu}\sin\varphi_\mu - s_\beta\sqrt{x_i}\sin\varphi_i)$$
$$+ \mathrm{Im}[(\mathbf{h}_U^\dagger \mathcal{Z}_U^\dagger)_{3I}(\mathcal{Z}_U \hat{\mathbf{A}}_U \mathcal{Z}_Q)_{IJ}\mathcal{Z}_Q^{\dagger J3}]s_\beta(c_\beta\sqrt{x_\mu}\cos\varphi_\mu + s_\beta\sqrt{x_i}\cos\varphi_i) \quad (i=1,2),$$

$$\Gamma^{W}_{q,IJ}=s_\beta c_\beta\sqrt{x_\mu}\{\mathrm{Re}[(\mathbf{h}^\dagger_q\mathcal{Z}^\dagger_q)_{3I}(\mathcal{Z}_q\hat{\mathbf{A}}_q\mathcal{Z}_Q)_{IJ}\mathcal{Z}^{\dagger J3}_Q]\cos\varphi_\mu-\mathrm{Im}[(\mathbf{h}^\dagger_q\mathcal{Z}^\dagger_q)_{3I}(\mathcal{Z}_q\hat{\mathbf{A}}_q\mathcal{Z}_Q)_{IJ}\mathcal{Z}^{\dagger J3}_Q]\sin\varphi_\mu\},$$

$$\Gamma^{F}_{q,IJ}=s_\beta c_\beta\sqrt{x_\mu}\{\mathrm{Re}[(\mathbf{h}^\dagger_q\mathcal{Z}^\dagger_q)_{3I}(\mathcal{Z}_q\hat{\mathbf{A}}_q\mathcal{Z}_Q)_{IJ}\mathcal{Z}^{\dagger J3}_Q]\sin\varphi_\mu+\mathrm{Im}[(\mathbf{h}^\dagger_q\mathcal{Z}^\dagger_q)_{3I}(\mathcal{Z}_q\hat{\mathbf{A}}_U\mathcal{Z}_Q)_{IJ}\mathcal{Z}^{\dagger J3}_Q]\cos\varphi_\mu\},$$

$$\Gamma^{V,i}_{D,IJ}=-\mathrm{Re}[(\mathbf{h}^\dagger_D\mathcal{Z}^\dagger_D)_{3I}(\mathcal{Z}_D\hat{\mathbf{A}}_D\mathcal{Z}_Q)_{IJ}\mathcal{Z}^{\dagger J3}_Q]c_\beta(s_\beta\sqrt{x_\mu}\cos\varphi_\mu+c_\beta\sqrt{x_i}\cos\varphi_i)$$
$$+\mathrm{Im}[(\mathbf{h}^\dagger_D\mathcal{Z}^\dagger_D)_{3I}(\mathcal{Z}_D\hat{\mathbf{A}}_D\mathcal{Z}_Q)_{IJ}\mathcal{Z}^{\dagger J3}_Q]c_\beta(s_\beta\sqrt{x_\mu}\sin\varphi_\mu-c_\beta\sqrt{x_i}\sin\varphi_i),$$

$$\Gamma^{E,i}_{D,IJ}=-\mathrm{Re}[(\mathbf{h}^\dagger_D\mathcal{Z}^\dagger_D)_{3I}(\mathcal{Z}_D\hat{\mathbf{A}}_D\mathcal{Z}_Q)_{IJ}\mathcal{Z}^{\dagger J3}_Q]c_\beta(s_\beta\sqrt{x_\mu}\sin\varphi_\mu-c_\beta\sqrt{x_i}\sin\varphi_i)$$
$$-\mathrm{Im}[(\mathbf{h}^\dagger_D\mathcal{Z}^\dagger_D)_{3I}(\mathcal{Z}_D\hat{\mathbf{A}}_D\mathcal{Z}_Q)_{IJ}\mathcal{Z}^{\dagger J3}_Q]c_\beta(s_\beta\sqrt{x_\mu}\cos\varphi_\mu+c_\beta\sqrt{x_i}\cos\varphi_i),$$

$$\mathcal{X}^{R}_{U,IJK}=s^2_\beta\mathrm{Re}[\mathcal{Z}^{\dagger 3I}_U(\mathcal{Z}_U\hat{\mathbf{A}}_U\mathcal{Z}_Q)^{IJ}(\mathcal{Z}_U\hat{\mathbf{A}}_U\mathcal{Z}_Q)^{\dagger JK}\mathcal{Z}^{\dagger K3}_U],$$

$$\mathcal{X}^{A}_{U,IJK}=s^2_\beta\mathrm{Im}[\mathcal{Z}^{\dagger 3I}_U(\mathcal{Z}_U\hat{\mathbf{A}}_U\mathcal{Z}_Q)^{IJ}(\mathcal{Z}_U\hat{\mathbf{A}}_U\mathcal{Z}_Q)^{\dagger JK}\mathcal{Z}^{\dagger K3}_U],$$

$$\mathcal{X}^{S}_{q,IJK}=\chi_q\mathrm{Re}[(\mathbf{h}_U\mathcal{Z}_Q)^{3I}(\mathcal{Z}_q\hat{\mathbf{A}}_q\mathcal{Z}_Q)^{\dagger IJ}(\mathcal{Z}_q\mathbf{A}_q\mathcal{Z}_Q)^{JK}(\mathcal{Z}^\dagger_Q\mathbf{h}^\dagger_U)^{3I}]\quad(q=U,D,\ \chi_U=s^2_\beta,\ \chi_D=c^2_\beta),$$

$$\mathcal{X}^{B}_{q,IJK}=\chi_q\mathrm{Im}[(\mathbf{h}_U\mathcal{Z}_Q)^{3I}(\mathcal{Z}_q\hat{\mathbf{A}}_q\mathcal{Z}_Q)^{\dagger IJ}(\mathcal{Z}_q\hat{\mathbf{A}}_q\mathcal{Z}_Q)^{JK}(\mathcal{Z}^\dagger_Q\mathbf{h}^\dagger_U)^{3I}]\quad(q=U,D,\ \chi_U=s^2_\beta,\ \chi_D=c^2_\beta),$$

$$\mathcal{X}^{T}_{q,IJK}=\chi_q\mathrm{Re}[\mathcal{Z}^{3I}_Q(\mathcal{Z}_q\hat{\mathbf{A}}_q\mathcal{Z}_Q)^\dagger_{IJ}(\mathcal{Z}_q\hat{\mathbf{A}}_q\mathcal{Z}_Q)_{JK}\mathcal{Z}^{\dagger K3}_Q]\quad(q=U,D,\ \chi_U=s^2_\beta,\ \chi_D=c^2_\beta),$$

$$\mathcal{X}^{C}_{q,IJK}=\chi_q\mathrm{Im}[\mathcal{Z}^{3I}_Q(\mathcal{Z}_q\hat{\mathbf{A}}_q\mathcal{Z}_Q)^\dagger_{IJ}(\mathcal{Z}_q\hat{\mathbf{A}}_q\mathcal{Z}_Q)_{JK}\mathcal{Z}^{\dagger K3}_Q]\quad(q=U,D,\ \chi_U=s^2_\beta,\ \chi_D=c^2_\beta),$$

$$\mathcal{X}^{U}_{q,IJK}=\chi_q\mathrm{Re}[(\mathbf{h}^\dagger_q\mathcal{Z}^\dagger_q)_{3I}(\mathcal{Z}_q\hat{\mathbf{A}}_q\mathcal{Z}_Q)_{IJ}(\mathcal{Z}_q\hat{\mathbf{A}}_q\mathcal{Z}_Q)^\dagger_{JK}(\mathcal{Z}_q\mathbf{h}_q)_{K3}]\quad(q=U,D,\ \chi_U=s^2_\beta,\ \chi_D=c^2_\beta),$$

$$\mathcal{X}^{D}_{q,IJK}=\chi_q\mathrm{Im}[(\mathbf{h}^\dagger_q\mathcal{Z}^\dagger_q)_{3I}(\mathcal{Z}_q\hat{\mathbf{A}}_q\mathcal{Z}_Q)_{IJ}(\mathcal{Z}_q\hat{\mathbf{A}}_q\mathcal{Z}_Q)^\dagger_{JK}(\mathcal{Z}_q\mathbf{h}_q)_{K3}]\quad(q=U,D,\ \chi_U=s^2_\beta,\ \chi_D=c^2_\beta).\tag{D2}$$

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